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OAC Teacher In-Service Program

Provincial Report OAC Calculus Examination Review

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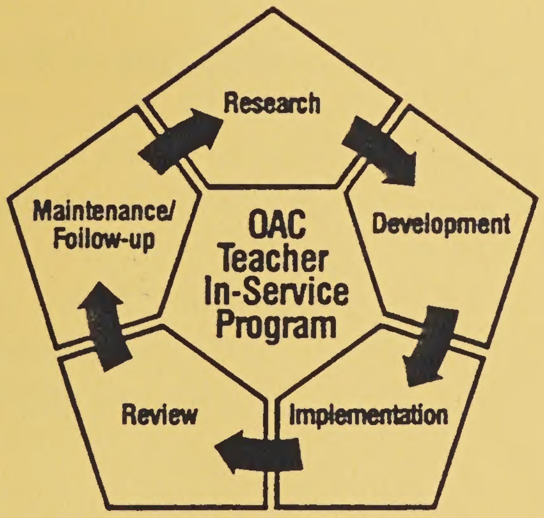

Ontario Ministry of Education
Program Implementation and Review Branch
18th Floor Mowat Block
900 Bay Street
Toronto, M7A 1L2

March 1991

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1 MODEL TO ACHIEVE PROVINCE-WIDE CONSISTENCY AT THE ONTARIO ACADEMIC COURSE LEVEL

1.1 PURPOSE OF THE REVIEW

In September 1989, the Ontario Academic Course (OAC) in Calculus replaced Grade 13 Calculus. The new course differs from the old in its emphasis on more varied examples of applications, less emphasis on algorithms for integration, the inclusion of some trigonometric identities formerly taught in Grade 13 Relations and Functions, and an increased emphasis on problem solving as a process component of the course.

The purpose of the OAC Calculus Examination Review was to determine the degree to which teachers have implemented a model to achieve province-wide consistency in the design of Calculus examinations and in the evaluation of student responses.

1.2 BACKGROUND

In the spring of 1986, a ministry-funded study of Grade 13 Calculus examinations from forty-four Ontario schools showed reasonable consistency in the length of examinations (adjusted for different question types). There was less consistency in the aids permitted (calculators, tables, formula sheets), and considerable diversity in the proportion of marks allocated to questions of different cognitive levels. It was also evident from the study that there was a wide diversity in the per cent of the final mark which was based on examination results. To increase the consistency of student evaluation for the new Ontario Academic Course in Calculus, a model based on the one used for OAC 1 English and OAC Visual Arts was developed. Criteria for designing an OAC Calculus examination and for evaluating student responses were established. The ministry anticipated that this model would produce a high degree of consistency and also improve classroom evaluation practices.

1.3 FEATURES OF THE OAC EXAMINATION MODEL

The Value of the Examination:

The curriculum guideline, *Mathematics Intermediate and Senior Divisions, 1985,* sets the value of formal examinations in any OAC in mathematics at not less than 40 per cent of the final mark.

To assist OAC Calculus teachers to develop province-wide examination consistency, a team of mathematics educators produced the handbook, "Criteria for the Design of the Examination and the Evaluation of Student Responses in OAC Calculus 1987". A series of workshops for OAC Calculus teachers was then held across the province to initiate implementation of the recommendations in the handbook. This handbook contained the following statements further clarifying the criteria for determining final marks.

- Whenever a fomal examination is set in OAC Calculus, no student should be exempted from writing it except in cases of illness or bereavement.
- In a specific class of OAC Calculus all students should be evaluated on the same components weighted in the same way.

Format of the Examination:

- The examination should be structured so that there is minimum switching among topics.
- Each section of the paper should be sequenced from easier questions to the more challenging ones.
- The examination should not ask students to make a choice among questions to answer.

Conditions for Administering the Examination:

- Permission to use calculators should be specifically granted on the examination paper.
- Measurement formulas should be included on the examination if required to answer a question.
- Key derivatives and/or common trigonometric identities should not be provided on the examination paper. The students should be expected to recall such derivatives and identities.

Selecting Examination Material and Designing Questions:

- The examination should reflect the relative importance placed on portions of the course as indicated by the time devoted to the topic and the emphasis given to it.
- Application and problem solving questions should represent at least thirty per cent of the marks allocated.
- There should be some relatively easy questions and some quite difficult questions. The majority of the questions should be of medium difficulty.

Evaluating Student Examination Responses:

- The marking scheme should not give undue emphasis to arithmetic calculations and algebraic simplifications. The majority of the marks should be allocated to the knowledge of concepts and procedures taught in OAC Calculus.
- Marks should be awarded for correct procedures, even if they reflect a previously made error.
- Marks should be deducted for missing steps only if such steps are considered essential for the proper development of the solution.
- Errors in form should be noted and deductions made for them. A maximum of ten per cent would be appropriate for the total marks deducted for form.

1.4 ADVANTAGES OF THE MODEL

The model ensures that all students are faced with similar questions, are evaluated on similar skills and have their responses assessed in similar ways. On the other hand, it allows a degree of flexibility. Teachers are then free to select specific questions representative of the material taught and individual schools can schedule examinations at a time convenient to them. The model also facilitates monitoring at school, board, regional, and provincial levels.

1.5 OAC TEACHER IN-SERVICE PROGRAM MODEL

The OAC In-Service Program in Calculus consists of the following five stages:

Research - Study of the examinations, marking schemes, and procedures for determining final marks from a sample of forty-four secondary schools across the province [spring 1986];

Development - Production of a handbook, "Criteria for the Design of the Examination and the Evaluation of Student Responses in OAC Calculus 1987", based on the research findings [fall 1986 and winter 1987];

Implementation - Half day in-service workshops for OAC Calculus teachers to initiate implementation of the handbook [spring-fall 1987];

Review - Collection and evaluation of each school's examination materials and the issuance of individual school results, jurisdiction reports, and a provincial report [spring-summer-fall 1990 and winter 1991];

Maintenance/Follow-Up - Maintenance of school programs identified as conforming to the policies and the recommended principles and practices; follow-up action, where necessary, by boards and schools to comply with ministry policy [winter-spring 1991 and continuing].

To assist mathematics teachers working to improve the standard of OAC Calculus examinations, an exemplary examination and marking scheme is included in the Provincial Report (see Appendices 1 and 2). The process model includes revision of the handbook based on input received from the field and, particularly, from the provincial review team that conducted the review.

2. EXAMINATION REVIEW

2.1 SCOPE OF THE REVIEW

In January 1990, the Ministry of Education informed school boards, secondary schools and inspected private schools that an examination review of the Ontario Academic Course Calculus (course code MCAOA) would be conducted, and requested that all schools offering the course submit the following materials to the Ministry of Education either through a Regional Office, or, in the case of summer schools, directly to the Program Implementation and Review Branch:

1. one OAC Calculus examination administered during the 1989-90 school year–the final examination in cases where more than one examination was written;
2. one detailed marking scheme for the examination;
3. three marked student responses representing high (80% plus), medium (60% to 79%), and low (59% and below) achievement;
4. a questionnaire completed by the mathematics department head or the OAC Calculus teacher.

Materials were requested from publicly-supported secondary schools (day school, night school and summer school programs) and inspected private schools. During the month of August, a Provincial Team of mathematics educators analyzed the submissions and prepared results for each school.

The Ministry's expectation is that the department head and teacher(s) responsible for the OAC Calculus will review their school results to determine to what degree the examination, marking scheme, student responses, and procedures for determining final marks, satisfy the policies stated in *Mathematics, Intermediate and Senior Divisions, 1985* and the expectations outlined in the handbook, "Criteria for the Design of the Examination and Evaluation of Student Responses in OAC Calculus 1987".

2.2 DATA GATHERING: THE SUBMISSION PROCESS

Step 1 - Informing Schools and Teachers Responsible

Early in 1990, a memorandum from each regional office informed all heads of mathematics or mathematics teachers responsible for OAC Calculus, of the ministry's intention to review their school's OAC Calculus examination, detailed marking scheme, and selected student responses. Enclosed with the memorandum was a questionnaire designed to collect details on the procedures used for the determination of final marks. Also enclosed was a draft of the checklist to be used for analysing the examination, the marking scheme, the evaluation of student responses, and the procedures for determining final marks. The checklist was derived from the handbook and the mathematics guideline. (See Appendix 3 for a copy of the School Results Examination Review OAC Calculus form.)

The memorandum instructed teachers to delete names from the marked papers to ensure anonymity of individual students.

Step 2 - Identifying Schools Offering an OAC Calculus Program

A list of all schools potentially offering OAC Calculus was compiled by the ministry and submitted to each regional office. Each regional office then undertook the collection of materials from each of the schools identified on the master list and any other schools that could be identified locally as offering OAC Calculus.

Step 3 - Setting Dates for the Submission of Examination Materials

Examinations from semestered day schools, non-semestered day schools, and night school programs were to be submitted by June 29, 1990. The submission date for summer school programs was the week of August 13, 1990.

Step 4 - Preparing and Forwarding the Examination Materials for Analysis

In July and early August of 1990, regional office representatives checked the submissions against their revised master lists, placed each school's examination materials in separate envelopes, and forwarded them to the Program Implementation and Review Branch for the provincial review team.

2.3 SELECTING THE REVIEW TEAM

The OAC Calculus review team represented all six educational regions of the province and consisted of twenty-seven OAC Calculus teachers and mathematics consultants from public, separate, and private school jurisdictions, a Ministry of Education official with mathematics teaching background, a professor of mathematics, and a professor of mathematics education.

The Manager of the OAC Teacher In-Service Program is Lloyd Thompson, Education Officer with the Ministry of Education, Program Implementation and Review Branch. The Project Leader for the Examination Review was David Alexander, professor of mathematics education at the Faculty of Education, University of Toronto.

2.4 THE REVIEW PROCESS

The review was conducted in Toronto over a twelve day period from August 13, to August 29, 1990.

During the initial two-day training period, the team members worked together to develop consistency in evaluating the four areas of analysis: the examination design, the detailed marking scheme, the evaluation of sample student responses, and the procedure for determining final marks.

Developing Consistency:

- Consistency in evaluation of the four areas of analysis was developed by reviewing the recommendations in the handbook and the policy stated in the guideline, identifying the criteria in the School Results forms, analyzing selected submissions, comparing the evaluations of individual team members, discussing differences in the evaluations, and establishing guidelines to minimize such differences.

- During the training period a list of comments, to identify specific strengths and weaknesses in the submissions, was developed by the team members. Throughout the review this list was expanded and refined to adjust to new situations.

- At the end of the training period, each reviewer was assigned an identification number. Working in pairs, they analyzed the submissions from each participating school. To assist in the consistency of analysis, partners were changed daily.

Analysis Procedure for the Pairs of Reviewers:

1. Select a package at random (avoiding schools for which either team member identifies a potential "conflict of interest").

2. Analyze the examination, complete the "Examination Format and Conditions" checklist, and add appropriate comments.

3. Analyze the examination, complete the "Detailed Marking Scheme" checklist, and add appropriate comments.

4. Analyze the evaluation of the three student responses, complete the "Evaluation of Student Responses" checklist, and add appropriate comments.

5. Analyze the response to the questionnaire, complete the "Procedures for Determining Final Marks" checklist, and add appropriate comments.

6. When a pair cannot agree on the appropriate completion of the checklist, the Project Leader is to be asked for a ruling.

7. Flag any material considered to be exemplary.

Time Required to Conduct a Complete Analysis:

When the school's submission consisted of one examination, three marked student responses and a completed questionnaire, the analysis took approximately one hour for a pair of reviewers to complete.

Final Check for Consistency:

After each team had completed the evaluation of a school's submission, the School Results form together with the submission was forwarded to a "checker" who reviewed the form to see that all sections were complete and that the information in each of the detailed analysis sections was consistent with that contained in the corresponding "Summary of Analysis" section.

3. RESULTS OF THE EXAMINATION REVIEW

3.1 PARTICIPATION RATES

Every public, separate and private secondary school offering a credit in OAC Calculus (course code MCAOA) was expected to submit one examination, a detailed marking scheme, three marked responses, and a completed questionnaire. This included submissions from all day schools, night schools, and summer schools within each jurisdiction. The total number of schools eligible to submit an OAC Calculus examination and materials was 833; the total number making a full or partial submission was 753. Thus the provincial participation rate in the examination review was 90 per cent.

FIGURE 1

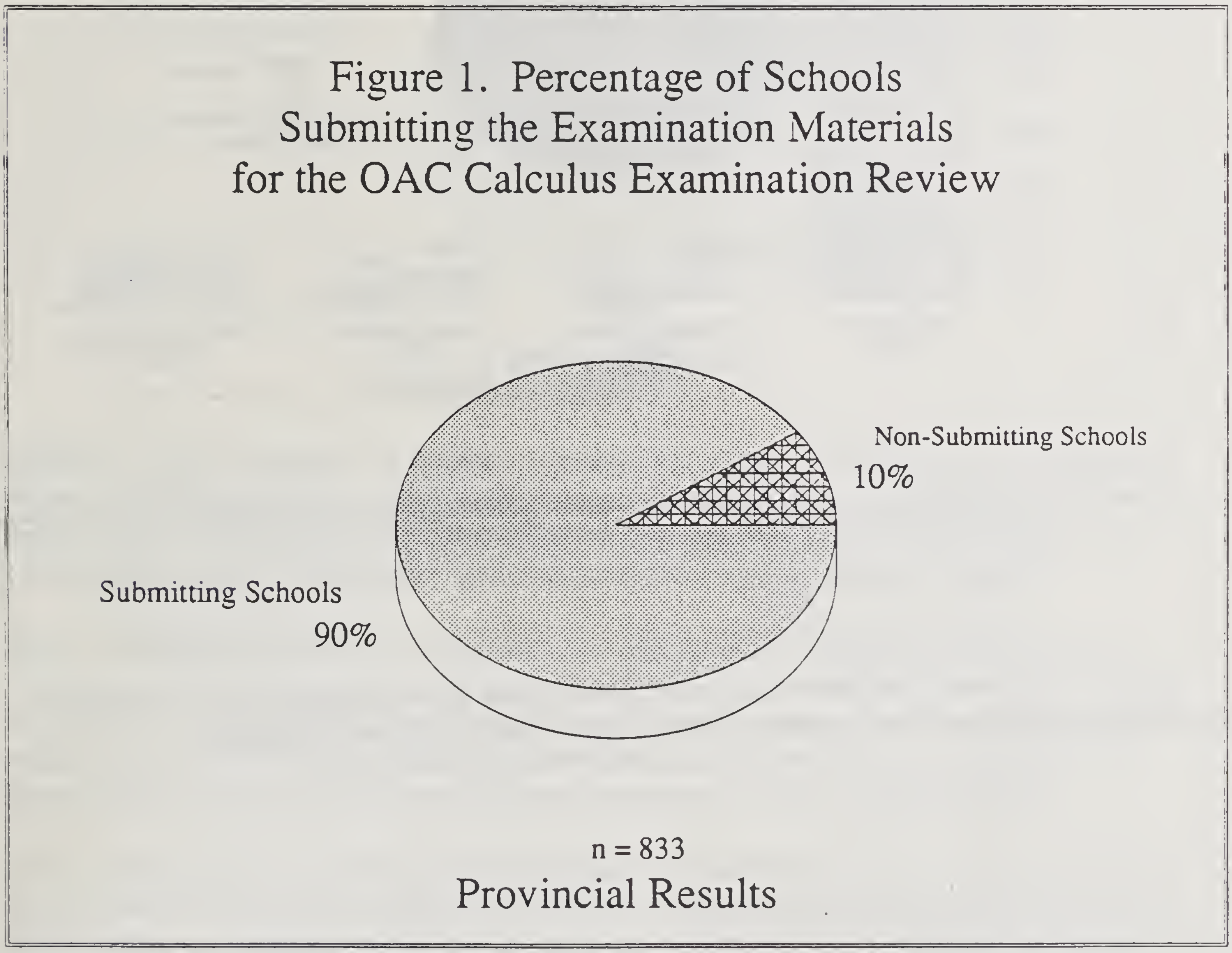


Figure 1. Percentage of Schools Submitting the Examination Materials for the OAC Calculus Examination Review

FIGURE 1a

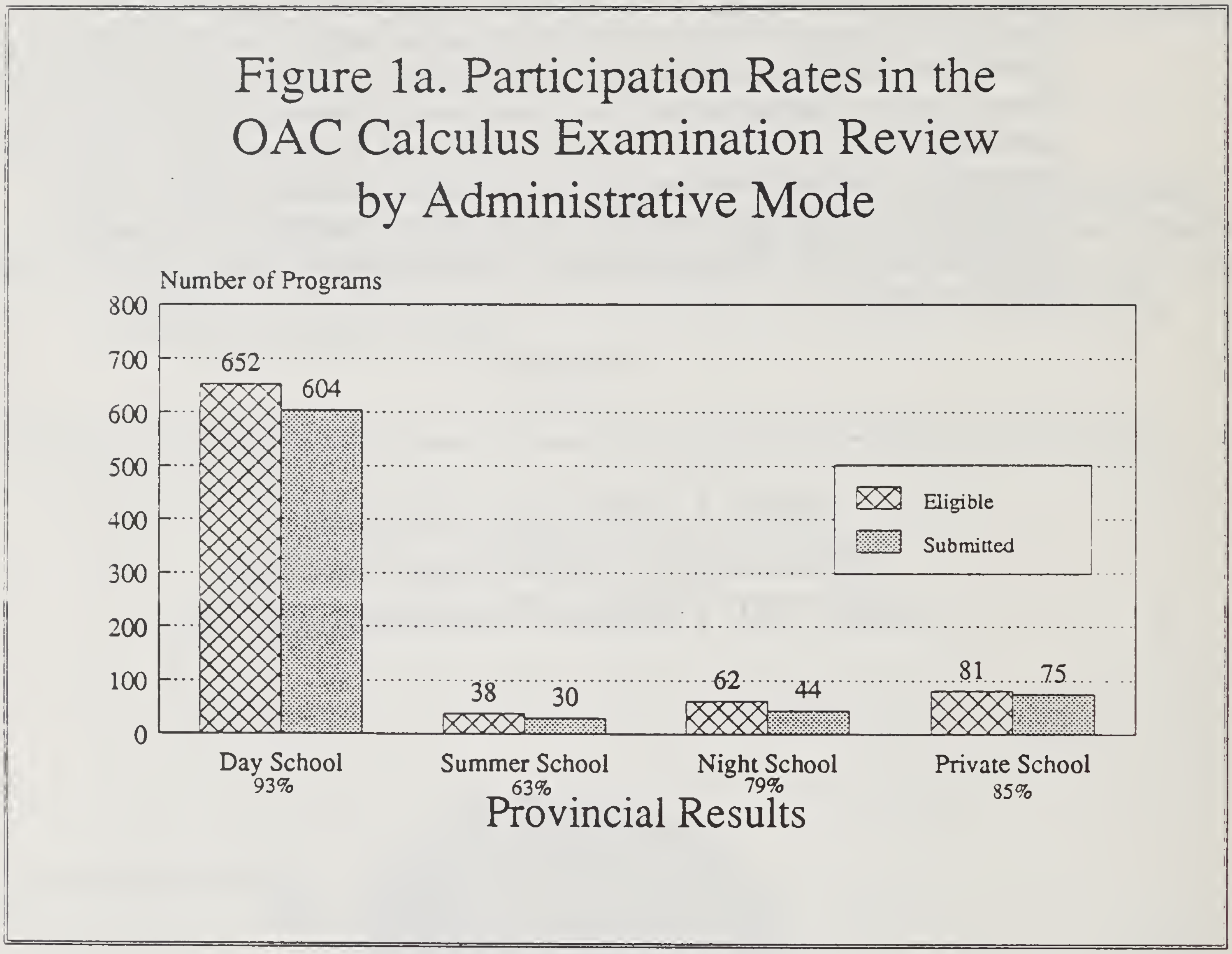
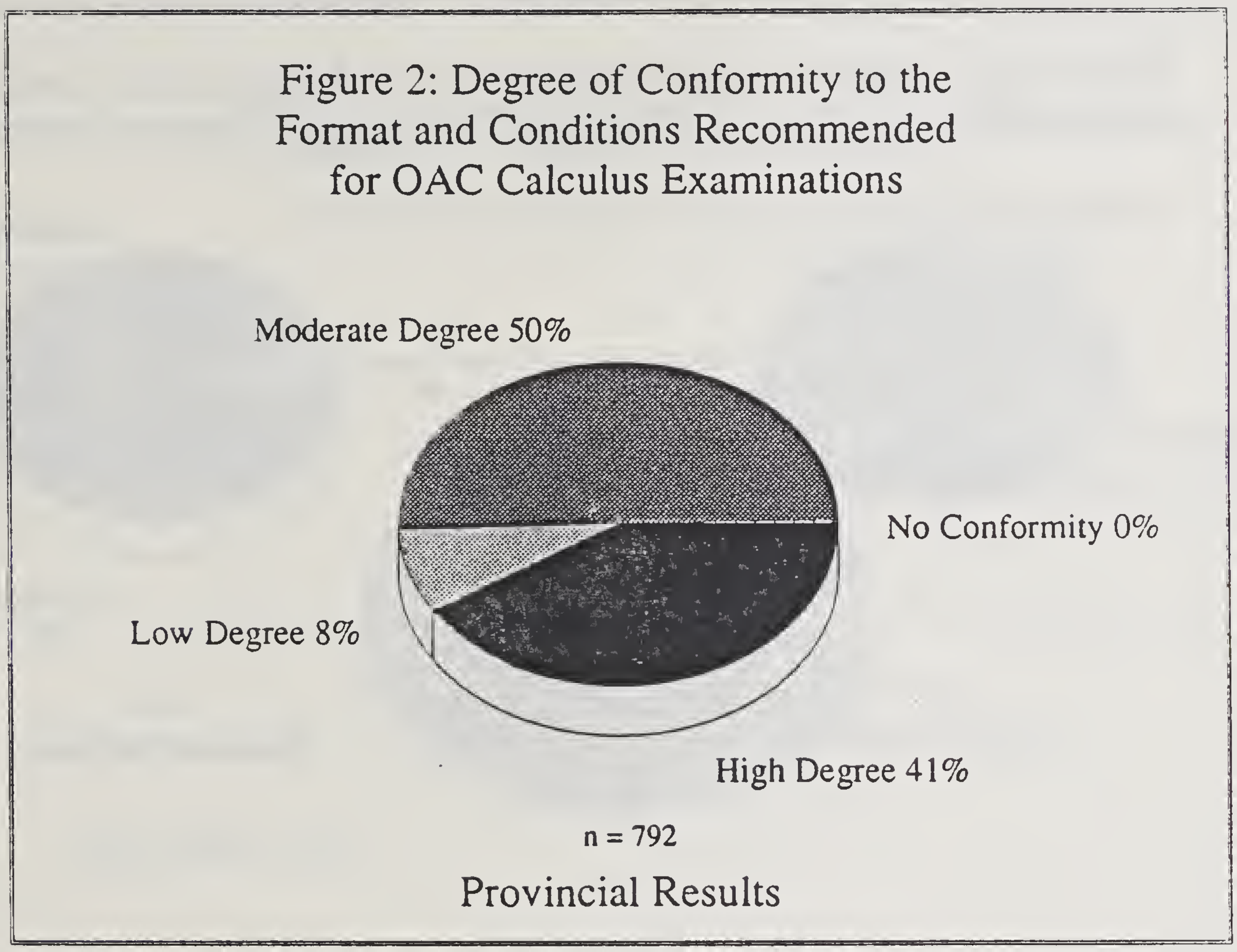


Figure 1a. Participation Rates in the OAC Calculus Examination Review by Administrative Mode

Figure 1a illustrates the provincial participation rates by administrative mode: publicly supported day schools, summer schools, night schools and private schools.

3.2 EXAMINATION FORMAT AND CONDITIONS

FIGURE 2



Figures 2 and 2a illustrate the degree of conformity of the OAC Calculus examinations submitted. Figure 2 shows the overall results, and Figure 2a the results by administrative mode. There were 792 submissions which included an examination. The numbers of examinations received from each administrative mode are indicated in Figure 2a.

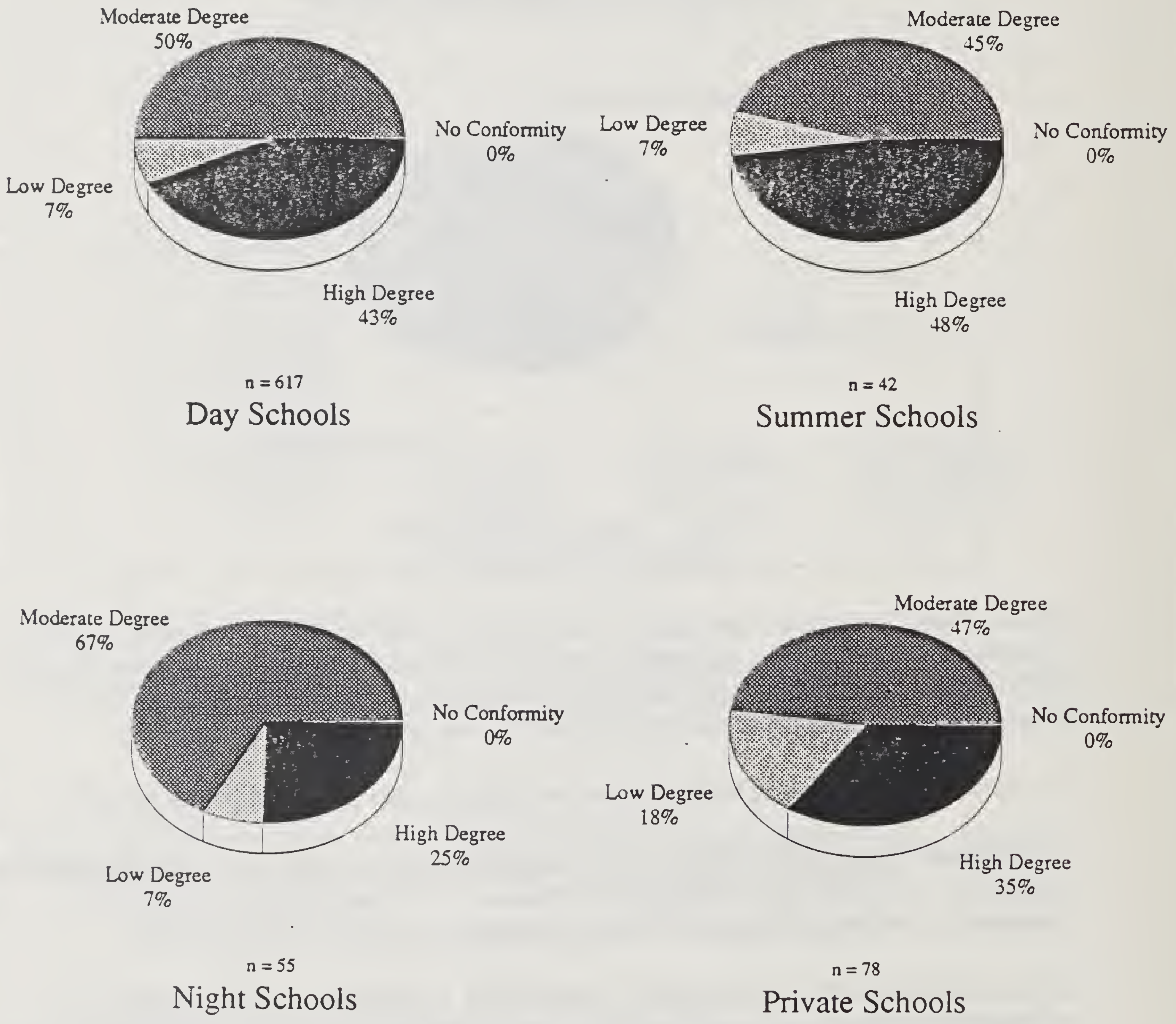
Due to rounding, the totals in some cases are slightly more or less than 100 per cent.

For purposes of this review, "conformity" was defined as the degree to which the examination adhered to the format and conditions recommended in "Criteria for the Design of the Examination and the Evaluation of Student Responses in OAC Calculus 1987".

Of the submissions 41 per cent of the examinations conformed to a high degree to the recommended format and conditions of an OAC Calculus examination, while 50 per cent conformed to a moderate degree, and 8 per cent conformed to a low degree or did not conform. (Only 2 submissions were identified as not conforming.)

FIGURE 2a

Figure 2a: Degree of Conformity to the Format and Conditions Recommended for OAC Calculus Exams by Administrative Mode



3.3 DETAILED MARKING SCHEME

FIGURE 3

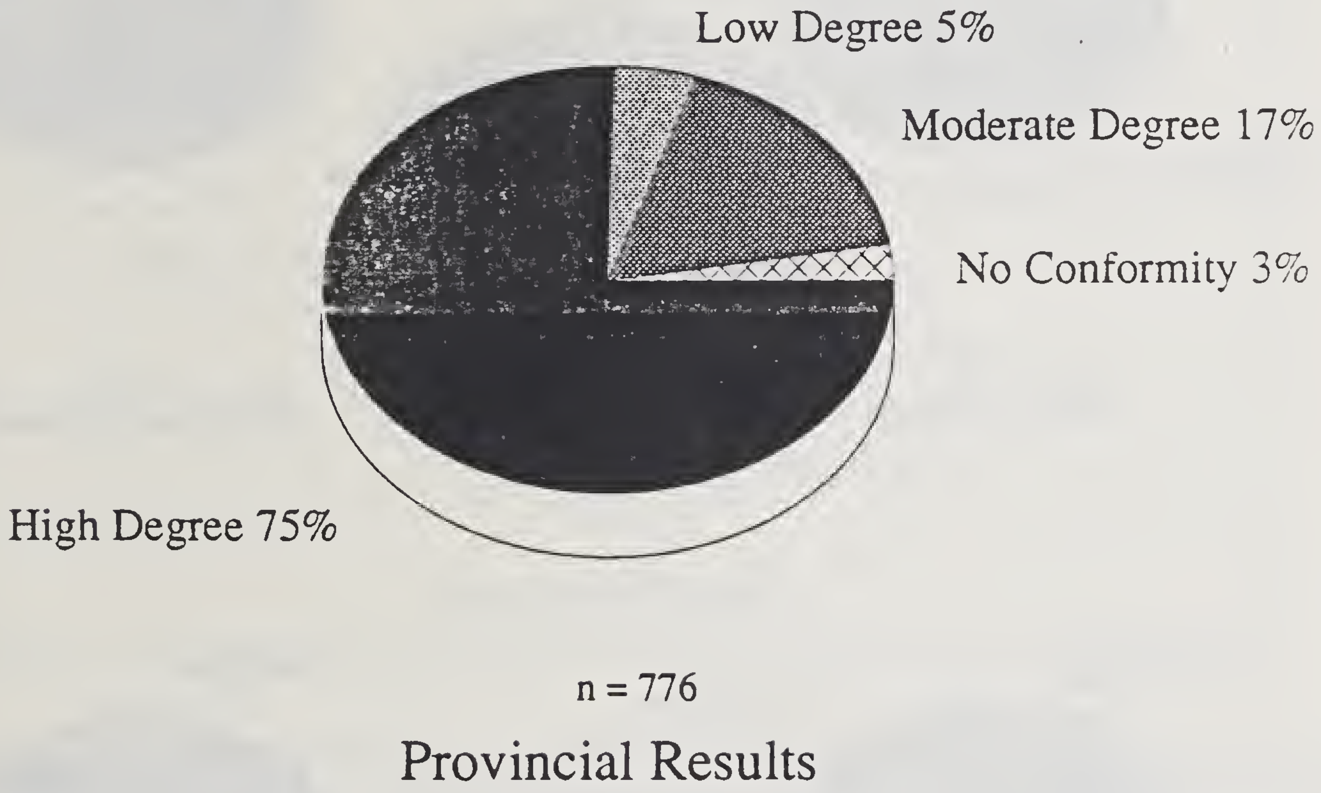


Figure 3: Degree of Conformity to the Marking Scheme Recommended for OAC Calculus Examinations

Figures 3 and 3a illustrate the degree of conformity of the detailed marking scheme to the principles recommended for an OAC Calculus examination. Figure 3 shows the overall results, and Figure 3a the results by administrative mode.

There were 776 submissions which included a detailed marking scheme. The numbers of detailed marking schemes received from each administrative mode are indicated on Figure 3a.

Of the submissions, 75 per cent conformed to a high degree, 17 per cent conformed to a moderate degree, while 8 per cent conformed to a low degree or did not conform.

FIGURE 3a

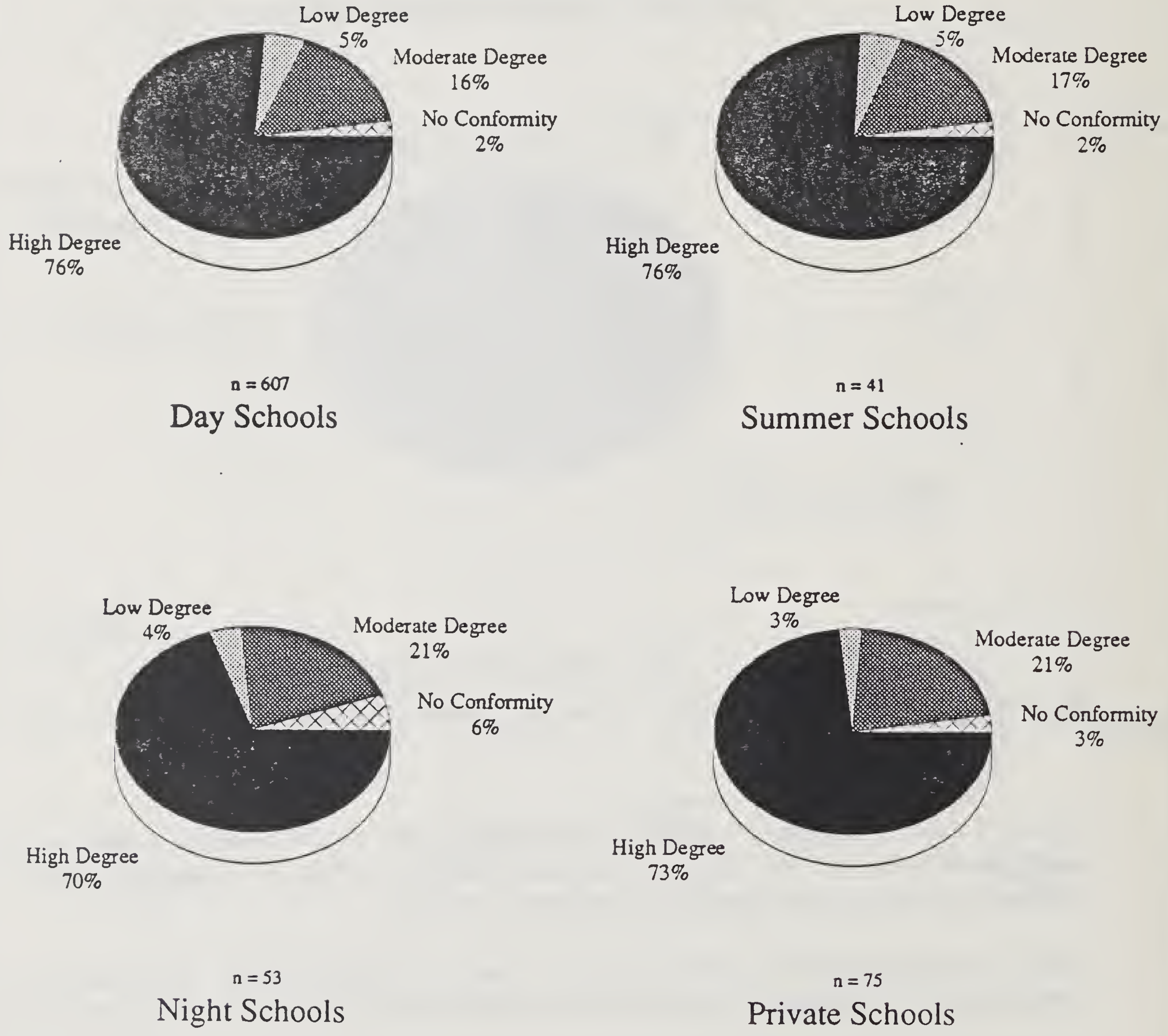


Figure 3a: Degree of Conformity to the Marking Scheme Recommended for OAC Calculus Exams by Administrative Mode

3.4 TEACHER MARKING OF STUDENT RESPONSES

FIGURE 4

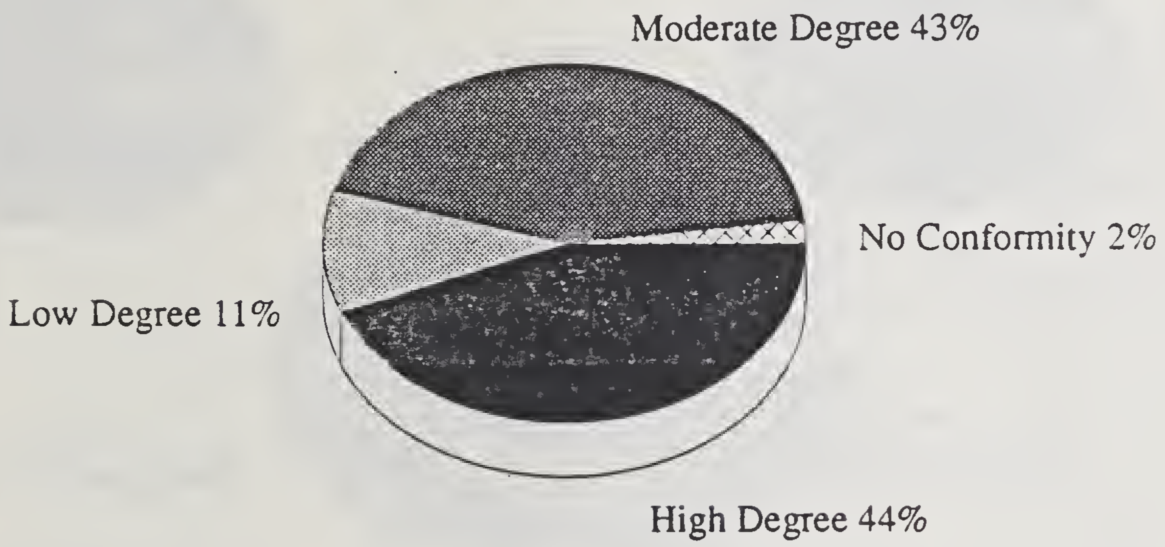


Figure 4: Degree of Conformity of Teacher Marking to Principles and Standards Recommended for Calculus Exams

n = 780

Provincial Results

Figures 4 and 4a illustrate the degree of conformity of the evaluation of student responses to the principles recommended for an OAC Calculus examination. Figure 4 shows the overall results, and Figure 4a the results by administrative mode.

There were 780 submissions which included marked student responses. The numbers of submissions from each administrative mode which included marked student responses are indicated on Figure 4a.

In 84 cases the submitted responses did not include a sample in the "high" range; 56 submissions did not include a sample in the "medium" range; 45 submissions did not include a sample in the "low" range.

Of the submissions, 44 per cent conformed to a high degree, 43 per cent conformed to a moderate degree, while only 13 per cent conformed to a low degree or did not conform.

FIGURE 4a

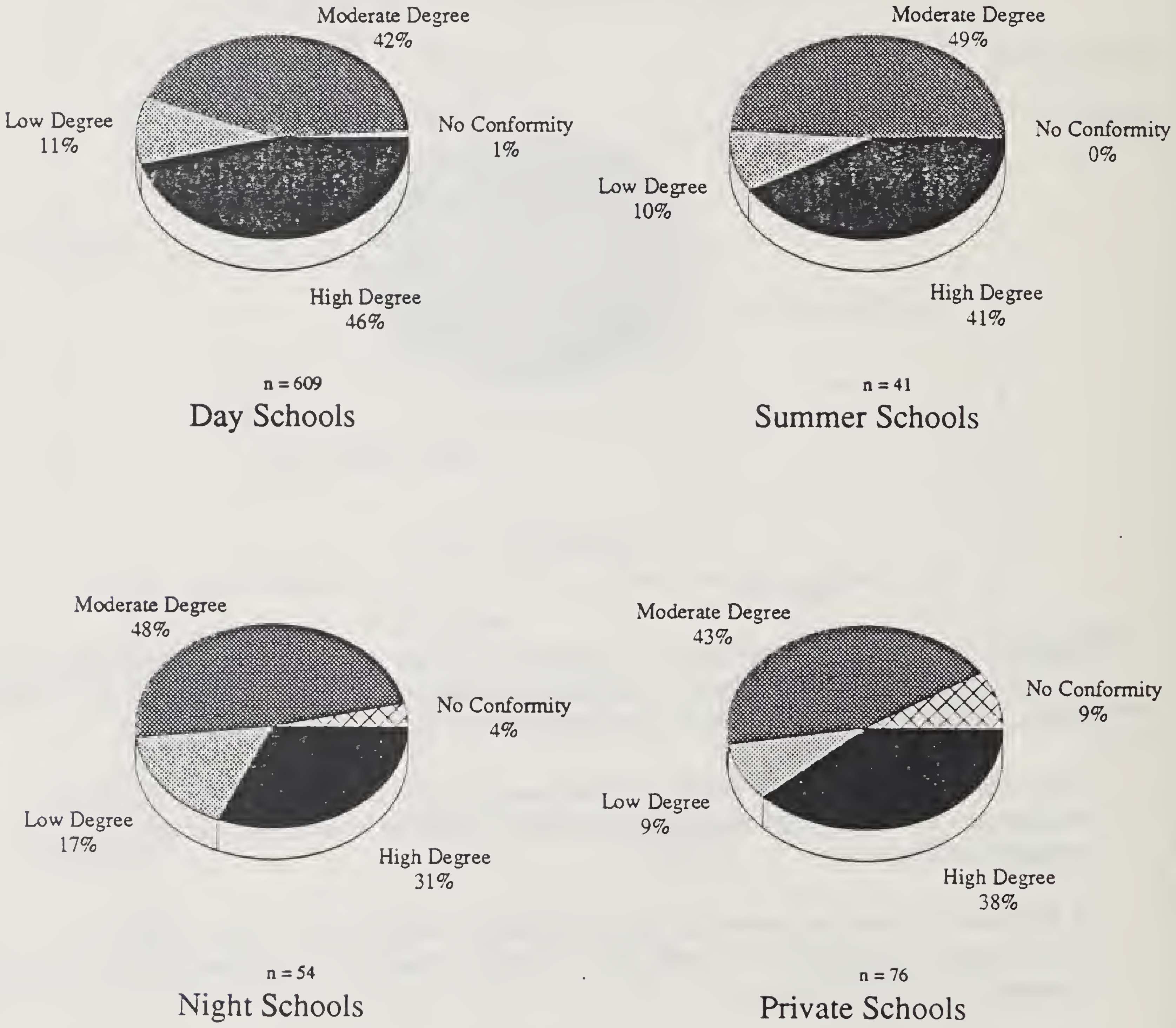
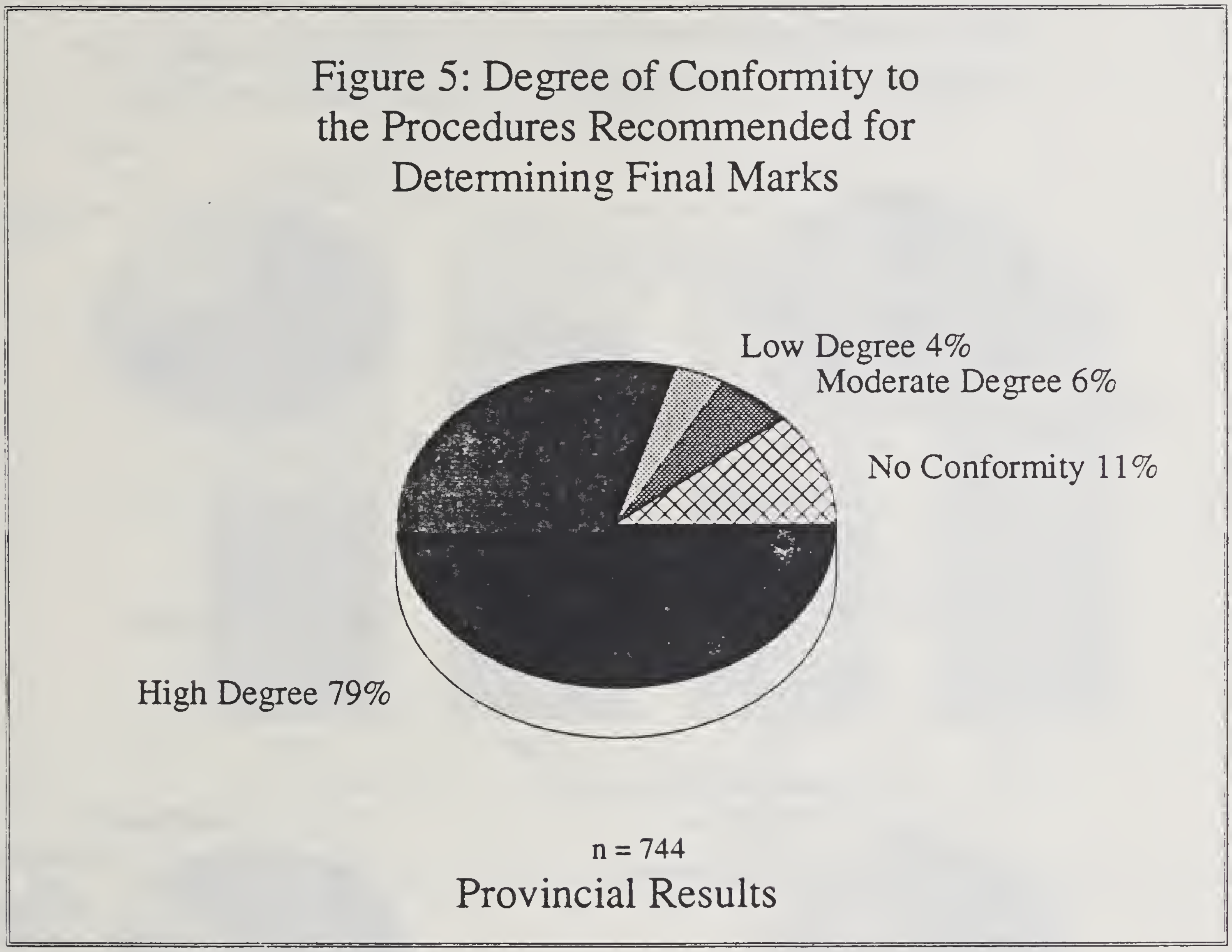


Figure 4a: Degree of Conformity of Teacher Marking of OAC Calculus Exams by Administrative Mode

3.5 PROCEDURES FOR DETERMINING FINAL MARKS

FIGURE 5



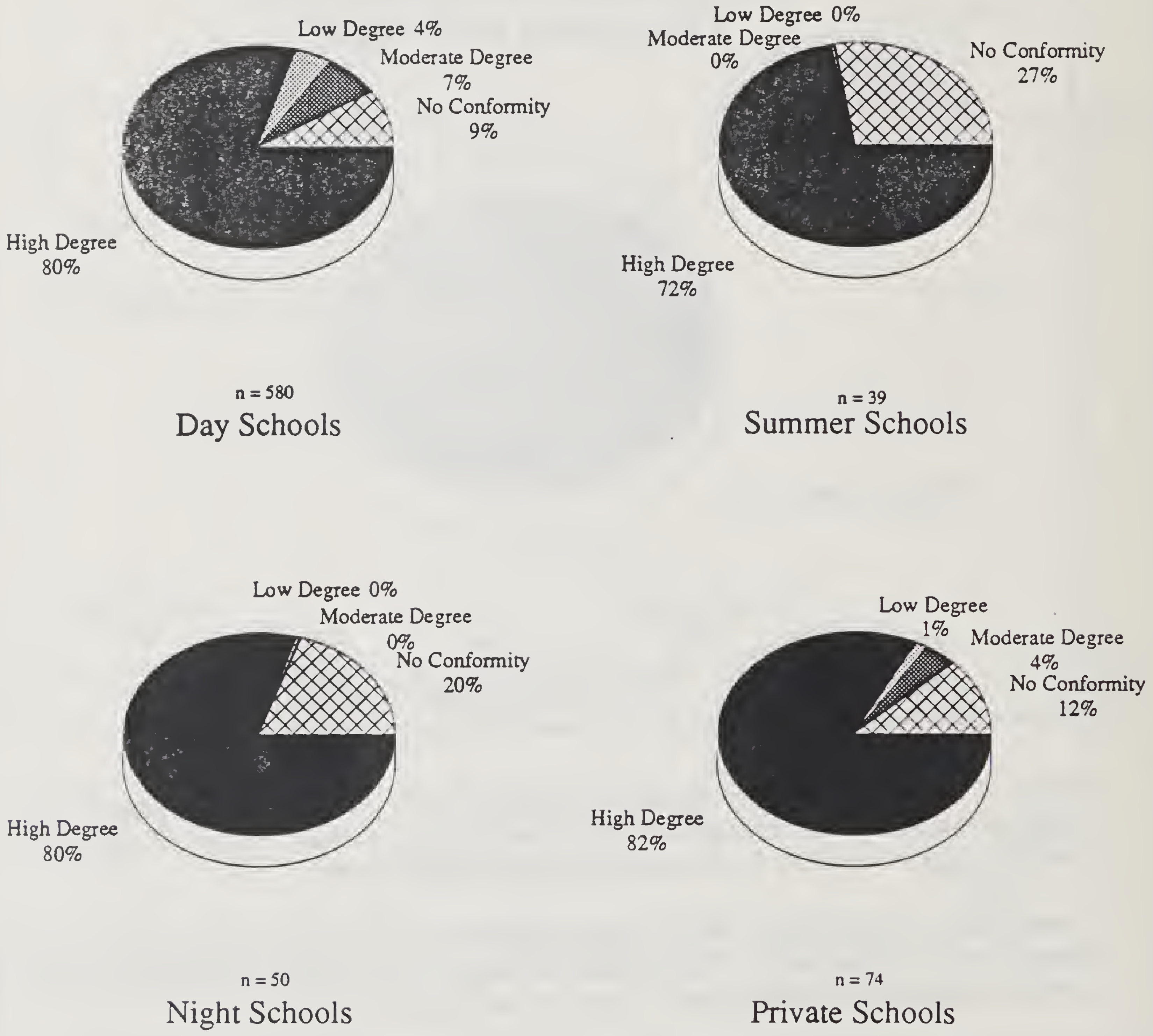
Figures 5 and 5a illustrate the degree of conformity to the procedures for determining final marks to the policy and recommended principles for OAC Calculus. Figure 5 shows the overall results, and Figure 5a the results by administrative mode.

There were 744 submissions which included information concerning procedures used for the determination of final marks. The numbers of submissions for each administrative mode which contained such information are indicated on Figure 5a.

Of the submissions, 79 per cent conformed to a high degree, 6 per cent conformed to a moderate degree, 4 per cent conformed to a low degree, and 11 per cent did not conform.

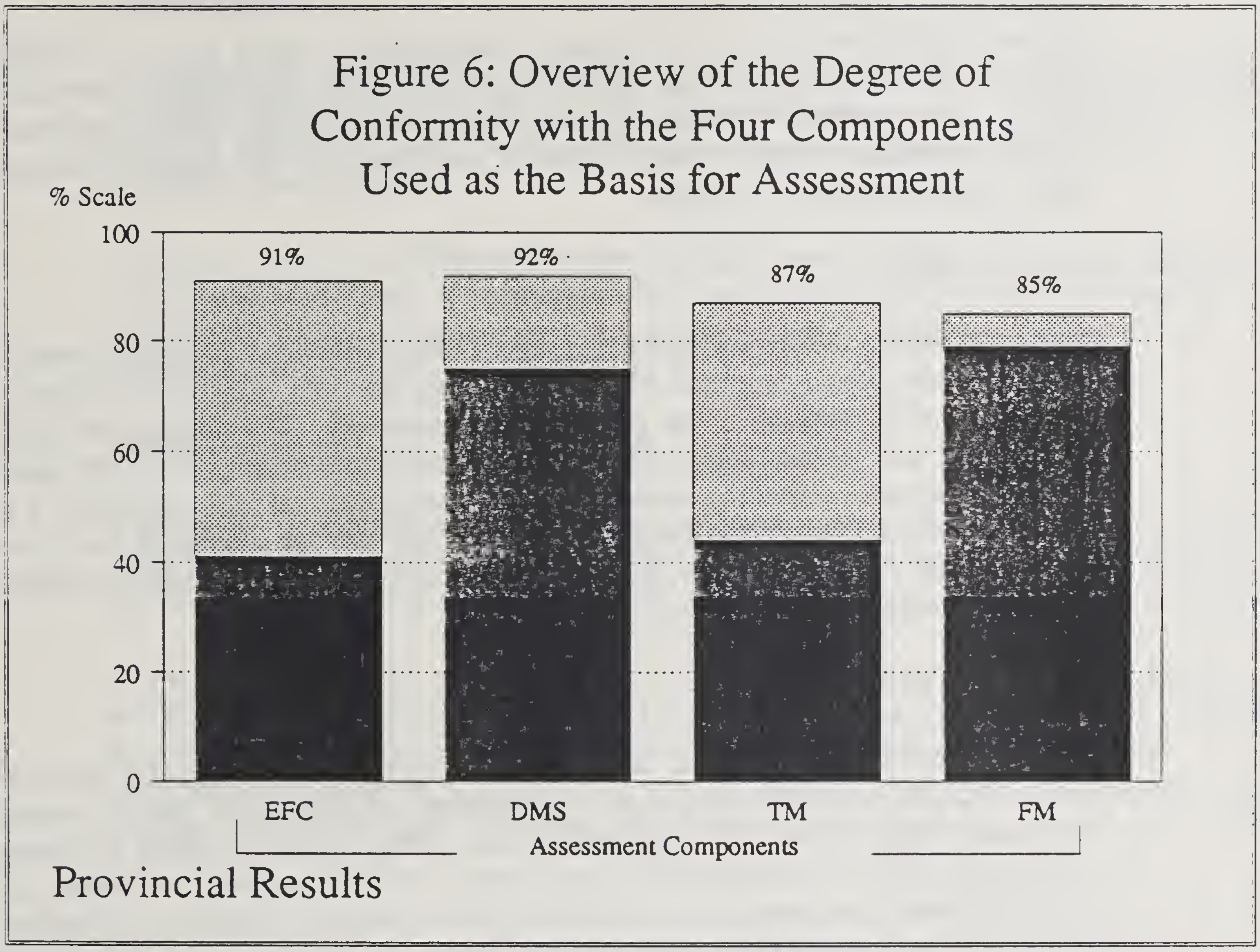
FIGURE 5a

Figure 5a: Degree of Conformity to the Procedures for Determining Final Marks by Administrative Mode



3.6 DEGREE OF CONFORMITY TO ALL CRITERIA

FIGURE 6



KEY to Figure 6

▨ Moderate Conformity ■ High Conformity

EFC - Examination Format and Conditions
DMS - Detailed Marking Scheme
TM - Teacher Marking of Student Responses
FM - Procedures for Determining Final Marks

Figure 6 illustrates the percentage of schools submitting examination materials that conformed to a moderate or high degree to each of the criteria used to assess all submissions. The first vertical bar indicates that 91 per cent conformed to a high or moderate degree to the Examination Format and Conditions as shown in Figure 2. Each of the following vertical bars with accompanying percentages is based on the data in Figures 3, 4, and 5.

4. CONCLUSION

4.1 FINDINGS OF THE REVIEW

The review has produced a clear picture of the degree to which the OAC Calculus Teacher In-Service Program has generated province-wide consistency in examination format, the evaluation of student responses, and procedures for determining final marks. It indicates there is a high degree of consistency in the design of examinations and in marking in OAC Calculus.

4.2 IN-SERVICE

The review has provided each participating school in the province with a detailed analysis of the degree to which it has implemented the OAC Calculus examination and evaluation model outlined in the guideline and handbook. The analysis provided to each school, and the exemplary material generated by the review, will offer direction for follow-up and in-service, where required. The review has also developed a group of trained professionals throughout Ontario, who can provide assistance for follow-up and in-service initiatives. A list of names and addresses of these resource people is provided in Appendix 4.

4.3 REVIEW FOLLOW-UP

Schools whose results confirm a high degree of conformity to the policy and principles for OAC Calculus examinations and determinations of final marks are encouraged to maintain the high standards established. Schools whose results indicate weaknesses in one or more of the four categories - the design of the examination, the design of the marking scheme, the marking of student responses, and the procedures for determining final marks - are expected to take corrective action to meet the ministry's expectations.

Where the school results indicate that any of the four categories "conforms to a low degree" or "does not conform" or there was "no submission", corrective action is to be taken during the current school year. A report of this corrective action is to be submitted to the principal and in the case of publicly supported schools, by the principal to the director of education. This information is then to be filed with the Regional Director of Education by June 30, 1991. In cases where the corrective action required is in Examination Format and Conditions or in the Detailed Marking Scheme, the report filed shall include an examination administered during the 1990-91 school year together with a detailed marking scheme.

Opportunities for in-service could be planned in conjunction with professional development days in which members of the provincial team are called upon as resource persons.

Included in this report are an exemplary examination (Appendix 1) and an exemplary detailed marking scheme (Appendix 2). These materials are included with a view to assisting teachers to meet the expectations for the design and marking of OAC Calculus examinations.

To further assist OAC Calculus teachers working toward consistency in standards of evaluation, the ministry is revising the handbook based on input received from the field and, in particular, from the review team members. Exemplary materials identified during the review will be included.

4.4 MONITORING

The ministry is continuing to monitor the evaluation practices for Ontario Academic Courses, to produce documents that outline the design and marking requirements for OAC examinations, and to conduct reviews of evaluation practices for selected OACs.

The impact of the OAC Teacher In-Service Program is encouraging, as evidenced by the widespread support for this collegial model, and by the consistency it has achieved in both the OAC 1 English and the OAC Calculus examinations. The ministry recognizes the importance of consistency in the evaluation of student achievement and believes that the program is successfully addressing this issue.

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APPENDIX 1

A SAMPLE OAC CALCULUS EXAMINATION

Note: The Examination has been reduced from legal size paper.

Calculus Examination
Carleton Roman Catholic
Separate School Board

MCA0A FINAL EXAMINATION

June, 1990

Student Copy

Name: ____

School: _______________________________________

Teacher's Name: ________________________________

Instruction for students:
Geometry sets and non-programmable calculators may be used. All solutions are to be written in the spaces provided. If additional space is required, use the back of the examination paper, indicating that you have done so.

Time: $2\frac{1}{2}$ hours

This examination consists of three sections:

Part A	Multiple Choice	20 marks
Part B	Short answer questions	36 marks
Part C	Complete solutions	64 marks
Total:		120 marks

Part A (20 marks) Write the letter which corresponds to the correct answer in the spaces provided. Each question is worth two marks.

1. If $f(x) = \dfrac{x^2 - x - 12}{x - 4}$, then $\lim\limits_{x \to 4} f(x)$ equals:

   A. $+\infty$ B. $-\infty$ C. 7 D. 0 1. ________

2. If $f(x) = \begin{cases} x^2, & x < 1 \\ 2, & x = 1 \\ -3x + 4, & x > 1 \end{cases}$, then which of the following statements is false?

   A. $\lim\limits_{x \to 1^-} f(x) = 1$ B. $\lim\limits_{x \to 1^+} f(x) = 1$

   C. f is continuous at x = 0 D. f is continuous at x = 1 2. ________

3. If f(x) and g(x) are functions such that $\lim\limits_{x \to 1} f(x) = -6$ and $\lim\limits_{x \to 1} g(x) = 7$, then which one of the following statements is false?

   A. $\lim\limits_{x \to 1} [f(x)\, g(x)] = 42$ B. $\lim\limits_{x \to 1} \dfrac{f(x)}{g(x)} = -\dfrac{6}{7}$

   C. $\lim\limits_{x \to 1} [\, f(x) + g(x)] = 1$ D. $\lim\limits_{x \to 1} [g(x)^2] = 49$ 3. ________

4. Evaluate $\lim\limits_{t \to 0} \dfrac{t}{\sin 5t}$

   A. 5 B. 0 C. 1 D. $\frac{1}{5}$ 4. ________

5. Which of the following describes all the local extrema of $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3$?

   A. a local maximum at x = 0 and a local minimum at x = 1
   B. a local minimum at x = 1
   C. a local maximum at x = 1
   D. a local maximum at x = 0 5. ________

6. If $f(x) = \cos x^3$, then an expression for $f'(x)$ is

   A. $-3x^2 \sin x^3$ B. $3x^2 \sin x^3$
   C. $-3\cos x^2 \sin x^2$ D. $-3\cos^2 x \sin x$ 6. ________

7. If $y = 3e^{-x}$, then $\dfrac{dy}{dx}$ is equal to

   A. $\frac{1}{3}e^{-x}$ B. $-3e^{\frac{1}{x}}$ C. $-3e^{-x}$ D. $3e^{-x}$ 7. ________

8. If $f(x) = 3x^3 - 4x^2 + 1$, which of the following statements is true?
   A. f has a local maximum at x = 0.
   B. f has a local minimum at x = 0.
   C. f has a point of inflection at (0,1).
   D. f is concave upward on the interval $(-\frac{1}{3}, \infty)$ 8. ________

9. The area bounded by $y = \cos x$ and the x-axis, $0 \le x \le \pi$ can be found by:

A. $2\int_0^{\frac{\pi}{2}} \cos x \, dx$ B. $-2\int_0^{\frac{\pi}{2}} \cos x \, dx$

C. $\int_0^{\pi} \cos x \, dx$ D. $-\int_0^{\pi} \cos x \, dx$ 9. ________

10. $f(x) = \ln(\sin x)$. An expression for $f'(x)$ is

A. $\frac{1}{\sin x \cos x}$ B. $\cot x$ C. $\frac{1}{\sin x}$ D. $e^{\sin x}$ 10. ________

Part B **(36 marks) Write your solutions in the spaces provided. Part marks may be awarded.**

1. (4 marks) On what interval(s) is the function $g(x) = x^2 - 2\ln x$ increasing?

2. (4 marks) Determine: $\int \frac{x-3}{(x^2-6x+4)^8} dx$

3. Determine the derivative of each function. Do not simplify.

a. (3 marks) $y = \left(\sqrt[3]{x^2 - 5x}\right)(1 - x)$

b. (2 marks) $s = \frac{1}{t} + \frac{t - 1}{t + 1}$

4. (5 marks)

If $\sin \theta = \frac{2}{3}$, $0 < \theta < \frac{\pi}{2}$, find the exact value of $\sin 4\theta$ without using a calculator.

5. (4 marks) Evaluate: $\lim_{x \to 0} \frac{\tan x + \sin x}{x \cos x}$

6. (4 marks) Determine y ' for $x^3 y^2 + 4x^2 = 5y^2$

7. (4 marks) Solve: $6 \sin^2 \theta + \cos 2\theta = 2,\ 0 \le \theta \le 2\pi$.

8. (3 marks) Two chemicals A and B react to form a product C. The amount of product C formed in time t seconds is given by $Q(t) = 2 - \frac{1}{t}$ (in grams) where $t \geq 1$. Calculate the average rate of production of C between 1 and 2 seconds.

9. (3 marks)
Determine the equation $y = F(x)$ that passes through (-2,4) and satisfies
$\frac{dy}{dx} = 6x^2 - 4x$.

Part C (64 marks) Write your solutions in the spaces provided.
Full marks will be awarded only for complete solutions.

1. (7 marks)
State and prove the product rule for derivatives. Let $F(x) = f(x)\,g(x)$.

2. (5 marks)

For $y = e^{x^2 - \cos x}$, determine the point(s) on the graph where the tangent line is parallel to the x-axis.

3. (8 marks) An airplane flies west at 150 m/s at an altitude of 1000 m. A searchlight on the ground in the same vertical plane as the airplane is kept focused on the airplane. What is the rate of revolution of the searchlight when the airplane is 500 m due east of the searchlight?

4. (7 marks) Air is blown into a spherical balloon so that its volume increases at a rate of 200 cm^3 /s. If the compression of the air is neglected, how fast is the radius increasing at the instant the diameter is 30 cm?
$(V = \frac{4}{3}\pi r^3)$

5. (9 marks) A malfunction in the anti-matter device causes the space-ship Enterprise to travel out of control according to the formula $s(t) = -\ln(t^2 - 2t + 1)$ I.G.U./ h , where one I.G.U. is equal to warp factor 1.

   a. Show that the ship was cruising along at its normal speed of 2 I.G.U / h when the malfunction occurred.

   b. Determine the initial acceleration.

   c. If the ship's speed reaches warp factor 20, it will disintegrate. How much time, (in minutes), do Spock and Captain Kirk have to save the ship?

6. (6 marks) Radioactive substances decompose by spontaneously emitting radiation. The rate of decay is proportional to the amount of the substance that has not yet disintegrated. A sample of Bismuth-210 decayed to 33% of its original mass after 8 days. Determine the half-life of this element.

7. (9 marks) Determine the area of the region between the curves defined by $y = x^3 - 6x$ and $y = 3x$.

8. (13 marks) Analyse the graph of $f(x) = \frac{1}{1 + x^2}$ under the headings indicated and then sketch the function.

a. intercepts	b. symmetry
c. asymptotes	d. local maximum and minimum points
e. points of inflection	

APPENDIX 2

A SAMPLE OAC CALCULUS DETAILED MARKING SCHEME

Note: The Marking Scheme has been reduced from legal size paper.

Detailed Marking Scheme
Aldershot High School
Halton Board of Education

June 1990 Time: 2.5 hours

MCA0A **CALCULUS** Page 1 of 8

Teacher: ____________________ Student: ____________________

Instructions: Calculators are permitted.
Trigonometry tables are available, upon request.
COMPLETE, ALGEGRAIC solutions, in good form, are required, to earn full marks.

Approximate
Marks

1. Determine each of these limits:

(a) $\lim_{x \to \infty} \left(\frac{1}{4}\right)^x = 0$ ①✓

1

(b) $\lim_{x \to 4} \frac{x^2 - 7x + 12}{x - 4} = \lim_{x \to 4} \frac{(x-3)(x-4)}{(x-4)}$ ①✓

2

$= \lim_{x \to 4} (x-3)$

$= 4 - 3$

$= 1$ ①✓

(c) $\lim_{x \to \infty} \frac{2x^2 - 3x}{x^2 + 5}$

2

$= \lim_{x \to \infty} \frac{2 - \frac{3}{x}}{1 + \frac{5}{x^2}}$ ①✓

$= \frac{2 - 0}{1 + 0}$

$= 2$ ①✓

(d) $\lim_{n \to 0} \frac{\sqrt{x+n} - \sqrt{x}}{n}$

4

$= \lim_{n \to 0} \frac{(\sqrt{x+n} - \sqrt{x})(\sqrt{x+n} + \sqrt{x})}{n(\sqrt{x+n} + \sqrt{x})}$ ①✓

$= \lim_{n \to 0} \frac{x + n - x}{n(\sqrt{x+n} + \sqrt{x})}$ ①✓

$= \lim_{n \to 0} \frac{n}{n(\sqrt{x+n} + \sqrt{x})}$

$= \lim_{n \to 0} \frac{1}{\sqrt{x+n} + \sqrt{x}}$ ①✓

$= \frac{1}{2\sqrt{x}}$ ①✓

2. Determine y' and simplify. You are NOT expected to express the results in simplest factored form.

(a) $y = x^2 \cdot e^{3x}$

3 $y = uv$ where $u = x^2$ and $v = e^{3x}$

$y' = u'v + v'u$ where $u' = 2x$ and $v' = 3e^{3x}$ ①✓

$y' = 2xe^{3x}$ ①✓ $+ 3e^{3x}x^2$ ①✓

(b) $3xy - y^2 = 7$

4 $3(x'y + y'x)$ ①✓ $- 2yy' = 0$ ①✓

$3y + 3y'x$ ①✓ $- 2yy' = 0$

$(3x - 2y)y' = -3y$

$y' = \dfrac{-3y}{3x - 2y}$ ①✓

3. Determine y' and express the result using a SINGLE trigonometric function of x, not of $(\pi - x)$.

$y = \cos(\pi - x)$

3

Answer #1	Answer # 2
$y = \cos\pi \cos x + \sin\pi \sin x$ ①✓	$y = \cos\theta$ where $\theta = \pi - x$
$y = (-1)\cos x + (0)(\sin x)$	$\frac{dy}{d\theta} = -\sin\theta$ where $\frac{d\theta}{dx} = 0 - 1$
$y = -\cos x$ ①✓	$\frac{dy}{dx} = (-\sin\theta)(-1)$ ①✓
$y' = -(-\sin x)$	$y' = \sin(\pi - x)$
$y' = \sin x$ ①✓	$y' = \sin\pi \cos x - \cos\pi \sin x$ ①✓
	$y' = (0)(\cos x) - (-1)\sin x$
	$y' = \sin x$ ①✓

4. Use LOGARITHMIC DIFFERENTIATION to determine y'. Simplify your answer, but do not substitute for y.

$y = \left(\dfrac{\sec x}{1-x}\right)^4$

5 $\ln y = \ln\left(\dfrac{\sec x}{1-x}\right)^4$

$\ln y = 4$ ①✓ $\ln\left(\dfrac{\sec x}{1-x}\right)$

$\ln y = 4[\ln \sec x - \ln(1-x)]$ ①✓

$\ln y = 4\ln \sec x - 4\ln(1-x)$ ①✓ ①✓

$\frac{1}{y}y' = 4\left(\frac{1}{\sec x}\right)(\sec x \tan x) - 4\left(\frac{1}{1-x}\right)(-1)$

$y' = y\left[4\tan x + \frac{4}{1-x}\right]$ ①✓

5. Solve for all real values of x. Explain your reasoning if certain solutions are rejected.

(a) $e^{2x} + e^x - 6 = 0$ (b) $\ln x + \ln 3 = 2$

4 $(e^x)^2 + 1(e^x) - 6 = 0$

2 $(e^x + 3)(e^x - 2) = 0$ ✓

$e^x = -3$ OR $e^x = 2$ ✓

$\ln e^x = \ln(-3)$ OR $\ln e^x = \ln 2$

$x = \ln(-3)$ OR $x = \ln 2$

$\ln(-3)$ is undefined. ✓

$\therefore\ x = \ln 2$ ✓

$\ln(3x) = 2\ln e$

$\ln(3x) = \ln e^2$ ✓

$3x = e^2$

$x = \frac{e^2}{3}$ ✓

6. Prove that $\dfrac{\sin^2 A + \cos 2A}{2\cos A - \sin 2A} = \dfrac{1 + \sin A}{2\cos A}$

Each logical step must be shown to earn full marks.

4 $LS = \dfrac{\sin^2 A + 1 - 2\sin^2 A}{2\cos A - 2\sin A\cos A}$ ✓ ✓

$= \dfrac{1 - \sin^2 A}{2\cos A\,(1 - \sin A)}$ ✓

$= \dfrac{(1+\sin A)(1-\sin A)}{2\cos A\,(1-\sin A)}$ ✓

$= \dfrac{1+\sin A}{2\cos A}$

$= RS$

7. A model rocket is 350 metres high and rising at 15 m/s. Part of the rocket is released and falls to the ground.

Recall that the acceleration due to gravity is 10 m/s/s.

Determine the time, and velocity, at which the part strikes the ground, below. Explain fully.

7 Let $a = -10$ m/s/s

$v = \int(-10)\,dt$

$v = -10t + 15$ m/s ✓

$s = \int(-10t + 15)\,dt$

$s = -5t^2 + 15t + 350$ m ✓

$s = 0$ if $-5t^2 + 15t + 350 = 0$

$t^2 - 3t - 70 = 0$ ✓

$(t-10)(t+7) = 0$ ✓

$t = 10$ OR $t = -7$

$\rightarrow$ Since $t \geq 0$, ✓

$t = 10$ s

at $t = 10$ s,

$v = -10(10) + 15$

$v = -100 + 15$

$v = -85$ ✓

The part takes 10 s to fall to the ground. It strikes the ground while falling at 85 m/s. ✓

8. Determine each of these antiderivatives:

(a) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

5

$= \int \frac{\cos u}{\sqrt{x}}\,dx$ where $u = x^{\frac{1}{2}}$ ✓ and $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$\therefore \frac{du}{dx} = \frac{1}{2\sqrt{x}}$ ✓

$\therefore 2\sqrt{x}\,du = dx$

$= \int \frac{\cos u}{\sqrt{x}}\,2\sqrt{x}\,du$

$= 2\int \cos u\,du$ ✓

$= 2\sin u + C$ ✓ $\longrightarrow 2\sin\sqrt{x} + C$ ✓

(b) $\int_3^5 \frac{x}{x-2}\,dx$

6

$= \int_{x=3}^{x=5} \frac{x}{u}\,dx$ where $u = x - 2$ and $\frac{du}{dx} = 1$

$\therefore du = dx$

✓ Half mark for each of $u = x - 2$ and $x = u + 2$

$= \boxed{\int_{u=1}^{u=3}}$ ✓ $\frac{u+2}{u}\,du$

$= \int_1^3 \left[1 + 2\left(\frac{1}{u}\right)\right] du$ ✓

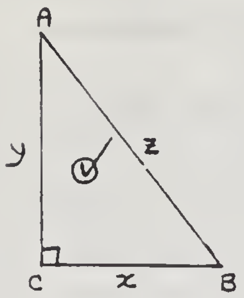
$= \int_1^3 du + 2\int_1^3 \frac{1}{u}\,du \longrightarrow = [u]_1^3 + 2\left[\ln|u|\right]_1^3$ ✓

$= (3-1) + 2(\ln 3 - \ln 1)$

$= 2$ ✓ $+ 2\ln 3$ ✓

9. Person A is 8 km north of corner C, and walking south at 4 kilometers per hour. Person B is 6 km east of C and walking east at 5 kilometers per hour. Find the rate of change in the distance between A and B. Are the people approaching or separating, at this instant?

7



$z^2 = x^2 + y^2$ ✓

$2zz' = 2xx' + 2yy'$ ✓

$zz' = xx' + yy'$

If $y = 8$ and $x = 6$,

$z^2 = 8^2 + 6^2$

$z^2 = 64 + 36$

$z^2 = 100$

$z = 10$ ✓

Let $10z' = (6)(5) + (8)(-4)$ ✓

$10z' = 30 - 32$

$10z' = -2$

$z' = -0.2$ ✓

A and B are approaching each other at 0.2 kilometres per hour. ✓

10. Consider the graph defined by $y = \dfrac{6}{x^2 - 2x - 3}$.

(a) Determine the equation(s) of all vertical asymptotes.

1

Let $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ OR $x = -1$ ✓

The vertical asymptotes are $x = 3$ and $x = -1$

(b) Determine the equation(s) of all horizontal asymptotes.

1

Let $y = \lim\limits_{x \to \pm\infty} \dfrac{6}{x^2 - 2x - 3}$

$y = \lim\limits_{x \to \pm\infty} \dfrac{6/x^2}{1 - 2/x - 3/x^2}$ $\longrightarrow$ $y = \dfrac{0}{1-0-0}$

The horizontal asymptote is $y = 0$. ✓

(c) Determine the x-intercept(s). If none exist, explain.

1

If $y = 0$, then $0 = 6$. As this is not true, there are no ✓ x-intercepts.

(d) Determine the y-intercept(s). If none exist, explain.

1

If $x = 0$, then $y = \dfrac{6}{0^2 - 2(0) - 3}$.

The y-intercept is -2 ✓

(e) Determine y', then use y' to find the extreme point(s).

You are NOT expected to prove that a point is a maximum or a minimum on the curve.

4

$y = \dfrac{u}{v}$ where $u = 6$ and $v = x^2 - 2x - 3$

$y' = \dfrac{u'v - v'u}{v^2}$ where $u' = 0$ and $v' = 2x - 2$

$y' = \dfrac{(0)(x^2-2x-3) - (2x-2)(6)}{(x^2 - 2x - 3)^2}$ ✓ ✓

$y' = \dfrac{12 - 12x}{(x^2 - 2x - 3)^2}$ $\longrightarrow$ $y' = 0$ if $12 - 12x = 0$

$x = 1$ ✓

At $x = 1$,

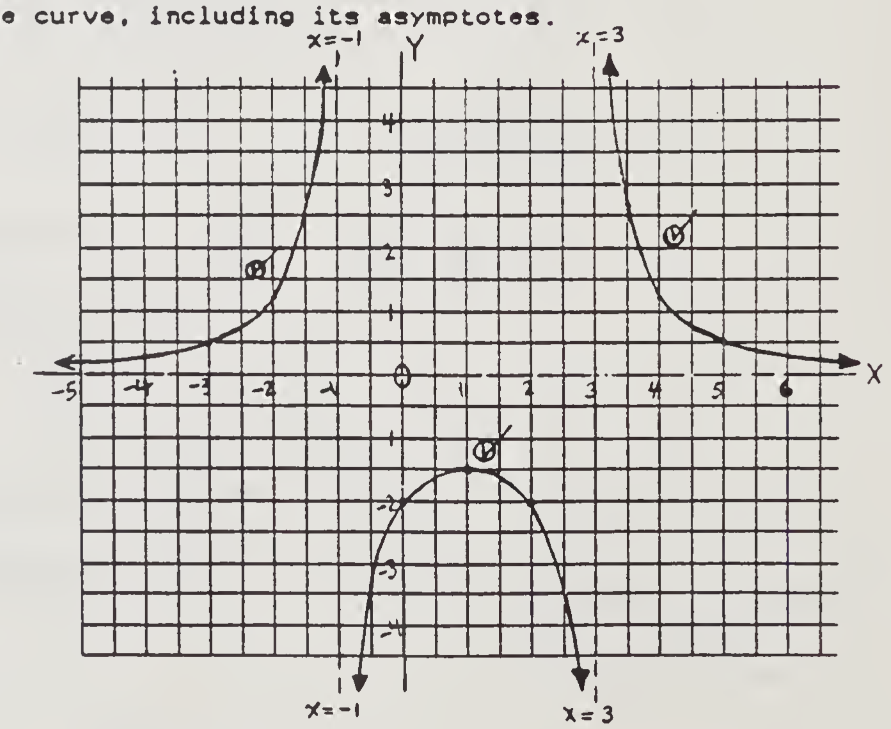
$y = \dfrac{6}{1-2-3}$

$y = -1.5$

The extreme ✓ point is $(1, -1.5)$.

(f) Sketch the curve, including its asymptotes.

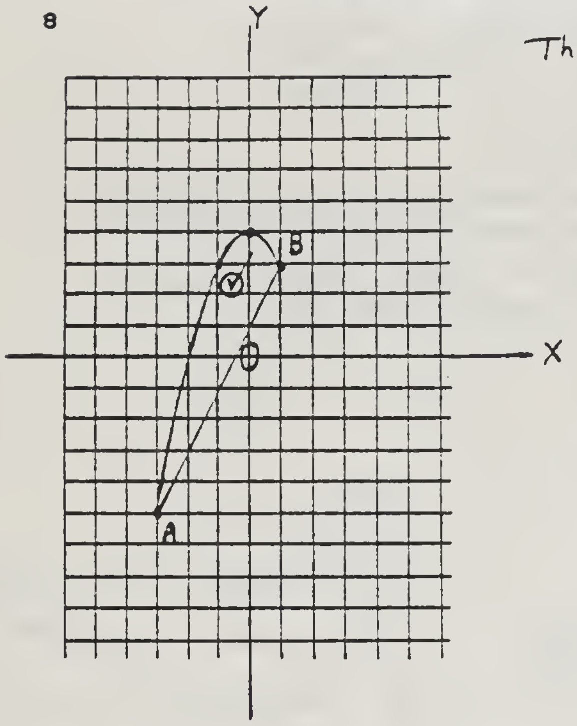
3



11. (a) Sketch the region enclosed by the curve $y = 4 - x^2$ and the chord joining A(-3,-5) to B(1,3).

(b) Determine the area of the region, given in part (a).

8



The Equation of AB is $\frac{y-3}{x-1} = \frac{3-(-5)}{1-(-3)}$ ✓

$$\frac{y-3}{x-1} = \frac{8}{4}$$

$$y-3 = 2(x-1)$$

$$y = 2x+1 \quad ✓$$

$$\text{Area} = \int_{-3}^{1}(4-x^2)\,dx - \int_{-3}^{1}(2x+1)\,dx \quad ✓$$

$$= \int_{-3}^{1}(4-x^2-2x-1)\,dx$$

$$= \int_{-3}^{1}(3-2x-x^2)\,dx \quad ✓$$

$$= \left[3x - x^2 - \tfrac{1}{3}x^3\right]_{-3}^{1} \quad ✓$$

$$= \left[3-1-\tfrac{1}{3}\right] - \left[-9-9+9\right] \quad ✓$$

$$= 1\tfrac{2}{3} + 9$$

$$= 10\tfrac{2}{3} \text{ square units} \quad ✓$$

12. Water leaks from a container such that the rate of change in the volume of water is proportional to the RECIPROCAL of the volume of water remaining in the container.

In the first 6 minutes, the volume decreases from 8 litres to 4 litres. Determine the total number of minutes that elapse before the container is EMPTY.

7

Let $\frac{dV}{dt} = k\left(\frac{1}{V}\right)$ ✓

$$V\,dV = k\,dt \quad ✓$$

$$\int V\,dV = k\int dt$$

$$\tfrac{1}{2}V^2 = kt + C \quad ✓$$

Let $\frac{1}{2}(8)^2 = k(0) + C$

$$32 = C \quad ✓$$

Let $\frac{1}{2}(4)^2 = k(6) + 32$ ✓

$$8 = 6k + 32$$

$$-24 = 6k$$

$$-4 = k \quad ✓$$

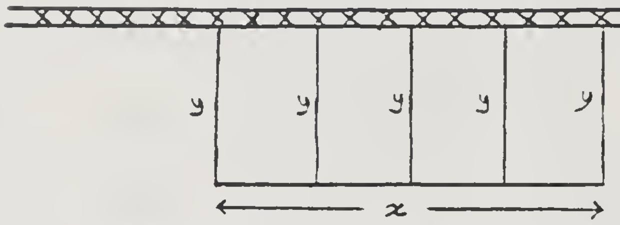
$\rightarrow$ Then $\frac{1}{2}V^2 = -4t + 32$

If $V = 0$, then $-4t + 32 = 0$

$$t = 8$$

The container will be empty in a total of 8 minutes. ✓

13. A rectangular plot of land is to be fenced, using an existing wall as one side. Furthermore, fencing is used to divide the plot into four regions, as shown.



①✓

If 400 metres of fence are available, find the maximum total area that can be enclosed, as shown, above.

Include a test for maximum or minimum, using a second derivative. Explain the test results, fully.

8

Let $x + 5y = 400$ ①✓

$x = 400 - 5y$

Let $A = xy$ ①✓

$A = (400 - 5y)(y)$

$A = 400y - 5y^2$ ①✓

$A' = 400 - 10y$

$A' = 0$ if $400 - 10y = 0$

$y = 40$ ①✓

If $y = 40$, $x = 400 - 5(40)$

$x = 400 - 200$

$x = 200$ ①✓ $\rightarrow$

$A'' = -10$

The graph of A versus y opens downwards. ①✓

If $x = 200$ and $y = 40$, then $A = 8000$

The maximum area enclosed is 8000 m^2 ①✓

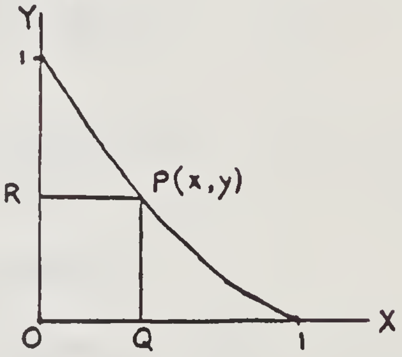
14. P(x,y) is a point, in the first quadrant, on the graph defined by

$$y = (1-x)^{\frac{3}{2}}.$$

Find the maximum AND minimum PERIMETER (NOT area) of the rectangle PQOR.

Include a test to justify your conclusions.

Graph of $y = (1-x)^{\frac{3}{2}}$



8

Let $z = 2x + 2y$ ①✓

$z = 2x + 2(1-x)^{3/2}$

$z' = 2 + 2\left[\frac{3}{2}(1-x)^{\frac{1}{2}}\right]\left[-1\right]$ ①✓

$z' = 2 - 3\sqrt{1-x}$ ①✓

$z' = 0$ if $2 - 3\sqrt{1-x} = 0$

$2 = 3\sqrt{1-x}$

Squaring, $4 = 9(1-x)$ ①✓

$4 = 9 - 9x$

$9x = 5$ ①✓

$x = \frac{5}{9}$ $\rightarrow$

x	$z = 2x + 2(1-x)^{3/2}$
0	2
$\frac{5}{9}$	$1\frac{19}{27}$ OR 1.7037037…
1	2

①✓

The maximum perimeter is 2 units. ①✓

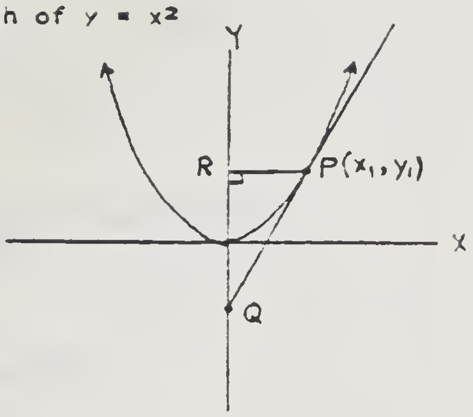
The minimum perimeter is $1\frac{19}{27}$ units. ①✓

15. The tangent at $P(x_1, y_1)$, on $y = x^2$, intersects the y-axis at Q, as shown.

(1) Find the coordinates of Q

(2) Find the area of Δ PQR. Express the result using x_1, only.

Graph of $y = x^2$



5 (1) Since $y = x^2$,

$y' = 2x$

The tangent at P is

$\frac{y-y_1}{x-x_1} = 2x_1$ ✓①

$y - y_1 = 2x_1 x - 2x_1^2$

If $x = 0$, then

$y = y_1 - 2x_1^2$

$\therefore$ Q is $(0, y_1 - 2x_1^2)$ ✓①

(2)

R is $(0, y_1)$

QR has length

$y_1 - (y_1 - 2x_1^2)$ ✓①

$= y_1 - y_1 + 2x_1^2$

$= 2x_1^2$ ✓①

RP is x_1 units long

$\Delta PQR = \frac{1}{2}(QR)(RP)$

$= \frac{1}{2}(2x_1^2)(x_1)$

$= x_1^3$ ✓① square units

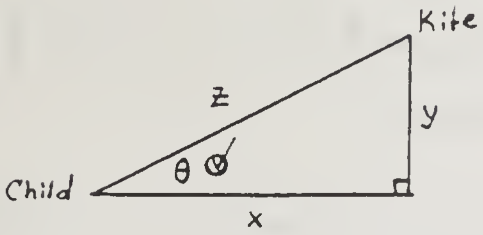
16. A child holds a string that is attached to a kite.

The string is 40 metres long, and growing in length by 0.2 metres per second.

The angle of elevation of the string is $\frac{\pi}{6}$, and growing by 0.01 radians per second.

Determine the rate of change in the height of the kite.

6



Answer #1

Let $\frac{y}{z} = \sin\theta$ ✓①

$y = z\sin\theta$

$y = uv$ where $u = z$ and $v = \sin\theta$

$y' = u'v + v'u$ where $u' = z'$ and $v' = (\cos\theta)(\theta')$

$y' = (z')(\sin\theta)$ ✓① $+ (\cos\theta)(\theta')(z)$ ✓①

Let $y' = (0.2)(\sin 30°) + (\cos 30°)(0.01)(40)$

$y' = (0.2)(0.5) + (0.866)(0.01)(40)$ ✓①

$y' = 0.1 + 0.3464$

$y' = 0.4464$

The kite i rising at the rate of 0.4464 m/s. ✓①

#16.

Answer # 2

✓ - Variables on Diagram

Let $\frac{y}{z} = \sin\theta$ ✓

$$\frac{y'z - z'y}{z^2} \checkmark = (\cos\theta)\,\theta' \checkmark$$

$$\frac{y'(40) - (0.2)(40\sin\frac{\pi}{6})}{(40)^2} = \left(\cos\frac{\pi}{6}\right)(0.01) \checkmark$$

$$40y' - 4 = 13.8564$$

$$40y' = 17.8564$$

$$y = 0.4464$$

The kite is rising at 0.4464 m/s. ✓

A Common Incorrect Answer – 4 marks out of 6 marks

✓ - Variables on Diagram

Let $\frac{y}{40} = \sin\theta$ $\frac{1}{2}$✓ [Half-mark]

$$\frac{y'}{40} = (\cos\theta)\,\theta' \checkmark$$

$y' = 40\left(\cos\frac{\pi}{6}\right)(0.01)$ $\frac{1}{2}$✓ [Half-mark]

$$y' = 0.3464$$

The kite is rising at 0.3464 m/s. ✓

APPENDIX 3

A SAMPLE COPY OF THE SCHOOL RESULTS

OAC Teacher In-Service Program

School Results OAC Calculus Examination Review

Ontario Ministry of Education
Program Implementation and Review Branch
18th Floor Mowat Block
900 Bay Street
Toronto, M7A 1L2

October 1990

SCHOOL RESULTS OF THE OAC CALCULUS EXAMINATION REVIEW

November 1990

The OAC Teacher In-Service Program

The OAC Teacher In-Service Program consists of the following five stages:

Research - Study of the examinations, marking schemes, and procedures for determining final marks from a sample of secondary school across the province.

Development - Production of a handbook, "Criteria for the Design of the Examination and the Evaluation of Student Responses in OAC Calculus".

Implementation - In-service workshops for the OAC teachers to implement the handbook.

Review - Collection and evaluation of each school's examination materials and the issuance of individual school results, jurisdictional reports, and a provincial report.

Maintenance/Follow-Up - Maintenance of school program identified as conforming to to the policies and the recommended principles and practices; follow-up action, where necessary, by boards and schools to comply with ministry policy.

Purpose of the OAC Calculus Examination Review

The OAC Calculus Examination Review has four major purposes:

1. to collect and analyse OAC Calculus examination materials from all schools offering the course to determine to what extent they satisfy provincial expectations;
2. to provide each school with a detailed analysis of its OAC Calculus examination, marking scheme, marking practices, and procedures for determining final marks;
3. to provide each jurisdiction with a report which summarizes the results for each school in the jurisdiction.
4. to produce a provincial report which summarizes the school results.

Background

In 1985, the Ministry of Education issued the curriculum guideline, *Mathematics, Intermediate and Senior Divisions*, with the expectation that implementation of the OAC Calculus course would commence no later than September 1989.

In January 1990 the Ministry of Education informed boards of education and inspected private schools that an examination review of the Ontario Academic Course, Calculus (course code MCAOA) would be conducted, and requested that all schools offering the course submit the following materials to the Ministry of Education either through a Regional Office, or, in the case of summer schools, directly to the Program Implementation and Review Branch:

1. one OAC Calculus examination administered during the 1989-90 school year - a final examination in cases where more than one examination was written;
2. one detailed marking scheme for the examination;
3. three marked student responses representing high (80% plus), medium (60% to 79%), and low (59% and below) achievement;
4. a questionnaire completed by the mathematics department head or OAC Calculus teacher.

Materials were requested from publicly-supported secondary schools (day school, night school and summer school programs) and inspected private schools. During the month of August, a Provincial Team of mathematics educators analyzed the submissions and prepared results for each school.

Responsibility of Department Heads and Teachers

The Ministry's expectation is that the department head and teacher(s) responsible for the OAC Calculus will review their school results to determine to what degree the examination, marking scheme, student responses, and procedures for determining final marks, satisfy the policies stated in *Mathematics, Intermediate and Senior Divisions, 1985* and the expectations outlined in the handbook, "Criteria for the Design of the Examination and the Evaluation of Student Responses in OAC Calculus".

Major Headings of the School Results

The analysis is organized under the headings:

1. Examination Format and Conditions
2. Detailed Marking Scheme
3. Evaluation of Student Responses
4. Procedures for Determining Final Marks

Action Based on School Results

Where the school results confirm a high degree of conformity, teachers should continue their efforts to maintain the standard established. Where the results indicate weaknesses in one or more of the categories of the design of the examination and/or the design of the marking scheme and/or the marking of student responses and/or the procedures for determining final marks, corrective action is required.

Where the school results indicate that any of the four categories "conforms to a low degree" or "does not conform" or there was "no submission", corrective action is to be taken during the current year. A report of this corrective action is to be submitted to the principal and, in the case of public supported schools, by the principal to the director of education. This information is then to be filed with the Regional Director of Education by June 30, 1991. In cases where the corrective action required is in Examination Format and Conditions or in the Detailed Marking Scheme, the report filed shall include an examination administered during the 1990-91 school year together with a detailed marking scheme.

Jurisdictional Report

Each director of education (or principal of a private school) will receive a **Jurisdictional Summary**, listing for each school in the jurisdiction the level of conformity under each of the headings.

Provincial Report

A **Provincial Report** of the results of the OAC Calculus Examination Review is being prepared and will be distributed to all boards of education and secondary schools. It will indicate the level of consistency of examinations and marking practices across the province. Individual boards and schools will not be identified in the provincial report.

Members of the Review Team

The review team of twenty-seven persons consisted of: OAC Calculus teachers from public, separate, and private schools, mathematics consultants, a Ministry of Education official with mathematics teaching background, a professor of mathematics, and a professor of mathematics education.

The Manager of the OAC Teacher In-Service Program is Lloyd Thompson, Education Officer with the Ministry of Education, Program Implementation and Review Branch. The Project Leader for the Examination Review was David Alexander, professor of mathematics education at the Faculty of Education, University of Toronto.

Training the Team

The review was completed in twelve days. During the initial two-day training period, the team members worked together to develop consistency in evaluating the four areas of analysis:

- Examination design
- Detailed marking scheme
- Evaluation of student responses
- Procedures for determining final marks.

Consistency in evaluation of the four areas of analysis was developed by reviewing the recommendations in the handbook and the policy stated in the guideline, identifying the criteria in the School Result forms, analyzing selected submissions, comparing the evaluations of individual team members, discussing differences in the evaluations, and establishing guidelines to minimize such differences.

During the training period a list of comments to identify specific strengths and weaknesses in the submissions was developed by the team members. Throughout the review this list was expanded and refined to adjust to new situations as they were identified.

The Review Process

At the end of the training period, each reviewer was assigned an identification number. Working in pairs, they analyzed the submissions from each participating school. To assist in the consistency of analysis, partners were changed daily.

Analysis Procedures for the Pairs of Reviewers

1. Select a package at random (avoiding schools for which either team member identifies a potential "conflict of interest").
2. Analyze the examination, complete the "Examination Format and Conditions" checklist, and add appropriate comments.
3. Analyze the marking scheme, complete the "Detailed Marking Scheme" checklist, and add appropriate comments.
4. Analyze the evaluation of the three student responses, complete the "Evaluation of Student Responses" checklist, and add appropriate comments.
5. Analyze the responses to the questionnaire, complete the "Procedures for Determining Final Marks" checklist, and add appropriate comments.
6. When a pair cannot agree on the appropriate completion of the checklist, the Project Leader is to be asked for a ruling.

If the school's submission consisted of one examination, a detailed marking scheme, three student responses, and a completed questionnaire, the analysis took an average of one hour for the pair of reviewers to complete.

The Final Check for Consistency

After each team had completed the evaluation of a school's submission, the School Results form together with the submission was forwarded to a "checker" who reviewed the form to see that all sections were complete and that the information in each of the detailed analysis sections was consistent with that contained in the corresponding "Summary of Analysis" section.

Results of the Review

The evaluation of each school's Calculus examination, marking scheme, evaluation of student responses, and procedures for determining final marks, is based on the professional opinion of experienced mathematics educators applying the criteria described in the handbook and outlined in the "School Results Examination Review OAC Calculus" printout circulated when the review was announced. Minor changes have been made in the form.

It is hoped that the analysis will be viewed as an opportunity for individual schools to review their procedures of evaluation and, where appropriate, to make changes which will produce consistency across the province in the evaluation of students in OAC Calculus. The results reported should not be used for school-to-school or teacher-to-teacher comparisons.

Revisions to the Handbook

To assist Calculus teachers working toward consistency in standards of evaluation, the ministry is revising the handbook based on input received from the field and, in particular, from the Review Team members. Exemplary materials identified during the review will be included.

Conclusion

It is hoped that this review will increase the level of consistency of student evaluation in OAC Calculus, while still maintaining the freedom of teachers to make professional decisions regarding evaluation based on local circumstances.

School Name
Board Name
School Address

School Number
Board Number
Region Code

School Results
Examination Review
OAC Calculus

Examination Format and Conditions

		YES	NO	N/A
1.1	Layout and typing are clear.			
1.2	The instructions to the student are clear.			
1.3	The examination instructions indicate that calculators may be used.			
1.4	The number of questions is appropriate to the time allowed for the examination and the types of questions it contains (e.g., multiple choice, short answers, full solution).			
1.5	The questions on different topics are in proportion to the relative importance of the topics on the course.			
1.6	There are some easy questions.			
1.7	There are some quite difficult questions.			
1.8	The majority of questions are of medium difficulty.			
1.9	The wording of the questions is clear.			
1.10	The examination is structured so that there is minimum switching among topics.			
1.11	The examination does not offer a choice between questions.			
1.12	Measurement formulas are provided as required.			
1.13	Key derivatives and common trigonometric identities are not provided on the examination.			

School Number	School Number Board Number

SUMMARY OF ANALYSIS

Examination Format and Conditions

This examination conforms to the format and conditions of an OAC Calculus Examination as described in "Mathematics, Intermediate and Senior Divisions, 1985" and "A Handbook for the Examination Component of Evaluation in the OAC Calculus".

1.14 to a high degree ________

1.15 to a moderate degree ________

1.16 to a low degree ________

1.17 does not conform ________

1.18 no submission ________

COMMENTS:

School Name	School Number Board Number

Detailed Marking Scheme

		YES	NO	N/A
2.1	The marking scheme is clear.			
2.2	More than 30% of the marks are awarded for application and problem solving.			
2.3	More than 50% of the marks are awarded for Calculus concepts and procedures.			

SUMMARY OF ANALYSIS

Detailed Marking Scheme

The marking scheme conforms to the principles recommended for an OAC Calculus Examination as described in "A Handbook for the Examination Component of Evaluation in the OAC Calculus."

2.4 to a high degree ________

2.5 to a moderate degree ________

2.6 to a low degree ________

2.7 does not conform ________

2.8 no submission ________

COMMENTS:

School Name	School Number Board Number

Evaluation of Student Responses

		YES	NO	N/A
3.1	The high quality student response submitted is within the range of high achievement in OAC Calculus.			
3.2	The medium quality student response submitted is within the range of medium achievement in OAC Calculus.			
3.3	The low quality student response submitted is within the range of low achievement in OAC Calculus.			
3.4	There is an appropriate awarding of marks for solutions not included in the marking scheme.			
3.5	Marks are awarded appropriately for correct procedures after an error.			
3.6	Marks are deducted for missing steps only when the steps are essential to a good solutions.			
3.7	Appropriate deductions are made for errors in form.			
3.8	Deductions for errors in form do not exceed 10% of the total marks for the examination.			

School Name	School Number Board Number

SUMMARY OF ANALYSIS

Evaluation of Student Responses

The evaluation of student responses conforms to the principles and standards recommended in "A Handbook for the Examination Component of Evaluation in the OAC Calculus."

3.9 to a high degree ________

3.10 to a moderate degree ________

3.11 to a low degree ________

3.12 does not conform ________

3.13 no submission ________

COMMENTS:

School Name School Number
Board Number

Procedures for Determining Final Marks

	YES	NO	N/A
4.1 There is a policy of no exemptions from examinations.			

4.2 For non-exempted students the examination component of the final mark is:

less than 40% ________

40% - 49% ________

50% - 59% ________

60% or more ________

N/A ________

4.3 For exempted students the examination component of the final mark is:

less than 40% ________

40% - 49% ________

50% - 59% ________

60% or more ________

N/A ________

4.4 The final mark is based on:
(Check all that apply)

examinations ________

unit tests ________

projects ________

individual study units ________

other ________

N/A ________

School Name	School Number Board Number

SUMMARY OF ANALYSIS

Procedures for Determining Final Marks

The determination of final marks conforms to the policy regarding the formal examination component as stated in "Mathematics, Intermediate and Senior Divisions, 1985".

4.5 to a high degree ________

4.6 to a moderate degree ________

4.7 to a low degree ________

4.8 does not conform ________

4.9 no submission ________

COMMENTS:

Ministry
of
Education
Ontario

Summary

Provincial Report OAC Calculus

Examination Review

OAC
Teacher
In-Service
Program

Consistent Standards of Evaluation

Since 1987, the Ministry of Education has worked with mathematics educators to develop criteria for student evaluation in the Ontario Academic Course (OAC) in Calculus. This is part of the OAC Teacher In-Service Program, an ongoing project to establish provincial standards for student evaluation at the OAC level. All OAC Calculus students must write examinations worth at least 40 per cent of their final mark. Consistent standards of evaluation across the province will assist universities to select students for admission.

Criteria

The criteria, published in a handbook distributed to all OAC Calculus teachers, dealt with three aspects of OAC Calculus examinations -- the examination format and design; the conditions for administering the examination; and the marking scheme. The handbook also outlined procedures for determining a student's final mark. The Ministry of Education organized workshops to help teachers put the handbook into practice.

Review

A review of OAC Calculus examinations took place in the 1989-90 school year. All public, separate, and private secondary schools offering OAC Calculus, including schools offering the course through night school or summer school, were asked to submit one examination (the final examination if the course included more than one examination); a detailed marking scheme; three marked student responses; and a description of the procedures used to determine final marks.

Participation

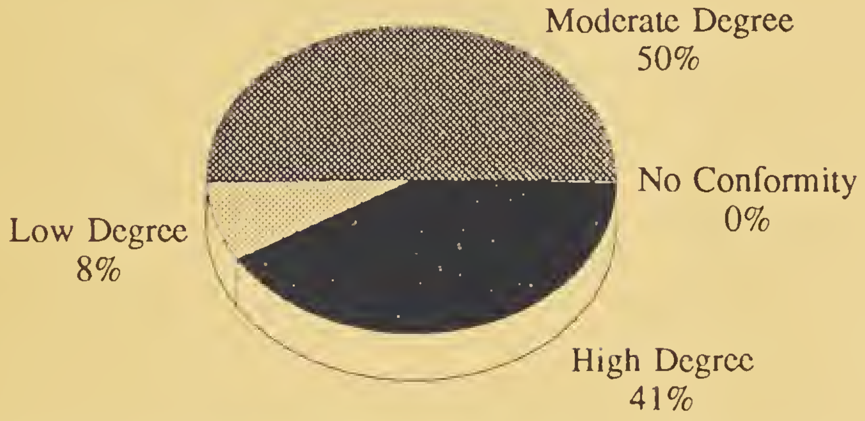
A total of **833 schools** were eligible to submit an OAC Calculus examination. This included schools that offered the course through night school or summer school. A total of **753** schools made a full or partial submission -- a participation rate of **90 per cent**.

Assessment

Each school's submission was reviewed by a team of OAC Calculus teachers and consultants, professors of mathematics education, and a Ministry of Education official with a background in teaching mathematics.

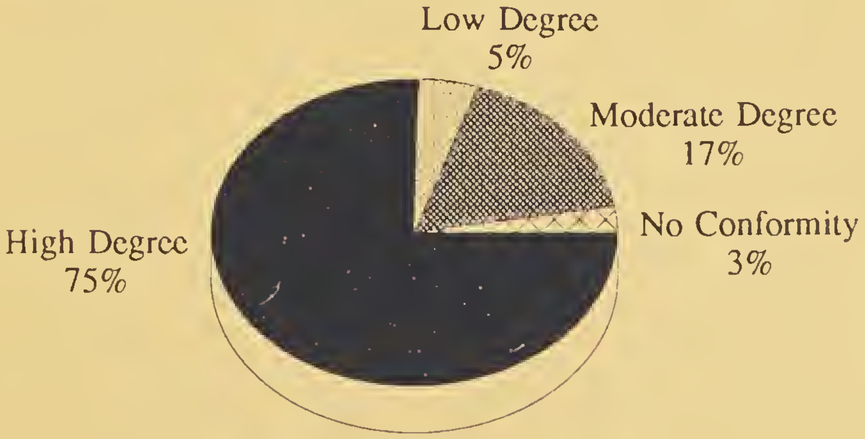
Results

Fomat and Conditions



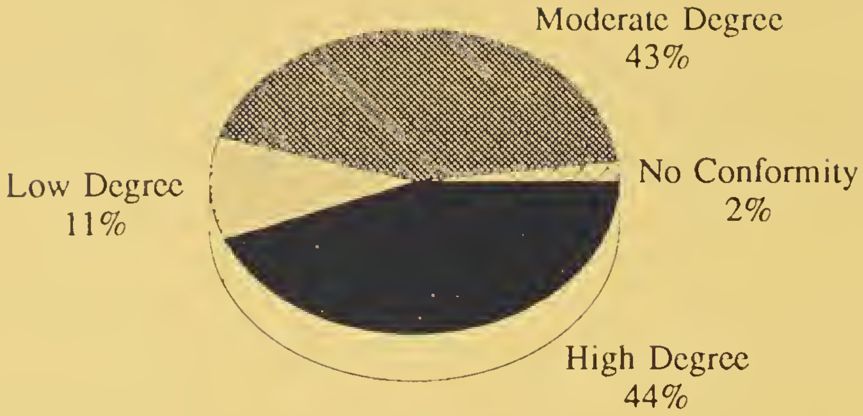
■ **91 per cent** of the schools conformed to the examination format and conditions to a high (41 per cent) or moderate (50 per cent) degree;

Marking Scheme



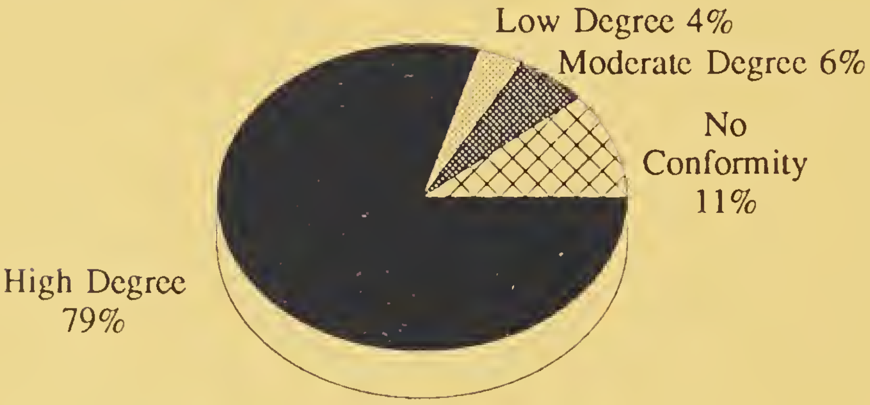
■ **92 per cent** of the schools conformed to provincial principles and standards for the detailed marking scheme to a high (75 per cent) or moderate (17 per cent) degree;

Marking of Responses



■ **87 per cent** of the schools provided marked student responses that conformed to provincial principles and standards to a high (44 per cent) or moderate (43 per cent) degree;

Determining Final Marks



■ **85 per cent** of the schools conformed to the procedures for determining final marks to a high (79 per cent) or moderate (6 per cent) degree.

Follow-up

Individual school and board reports have been distributed to principals and school board officials.

Schools whose examinations showed a high degree of conformity to the criteria are encouraged to maintain the high standards established.

Schools that did not submit examinations for review, or that demonstrated low conformity or no conformity to any of the criteria, will be required to take corrective action during the 1990-91 school year.

Conclusion

A large majority of mathematics teachers in Ontario are designing and marking OAC Calculus examinations in conformity to provincial standards.

APPENDIX 4

MEMBERS OF THE OAC CALCULUS REVIEW TEAM

OAC CALCULUS EXAMINATION REVIEW TEAM

Mr. Lloyd Thompson
Program Manager
Ministry of Education
Program Implementation & Review Branch
18th Floor Mowat Block
Toronto, Ontario
M7A 1L2
(416)965-8765

Dr. David Alexander
Program Leader
Faculty of Education
University of Toronto
371 Bloor Street West
Toronto, Ontario
M5S 2R7
(416)978-4068

Mr. Ian Anderson
Ancaster High and Vocational School
374 Jerseyville Road West
Ancaster, Ontario
L9G 3K8
(416) 648-4468

Mr. Glen Bonham
Streetsville Secondary School
72 Joymar Drive
Mississauga, Ontario
L5M 1G3
(416)826-1195

Mr. Gordon Burlison
Sir John A. MacDonald S.S.
130 York Street
Hamilton, Ontario
L8R 1Y5
(416)528-8363

Mr. Dan Charbonneau
St. Charles College
45 Spruce Street
Garson, Ontario
P0M 1V0
(705)693-2781

Mr. John Clarke
Northern Collegiate Institute & Vocational School
940 Michigan Avenue
Sarnia, Ontario
N7S 2B1
(519)542-5545

Ms Judy Crompton
Lincoln County Board
112 Oakdale Avenue
St. Catharines, Ontario
L2P 3J9
(416)685-1551 Ext 2228

Mr. Pierre Drouin
CSLF d'Ottawa-Carleton (catholique)
André Laurendeau E. S.
235 Avenue McArthur
Vanier, Ontario
K1L 6P3
(613) 746-8113

Mr. Russell Garrett
Co-ordinator Pure & Applied Science
Lakehead Board of Education
125 South Lillie Street
Thunder Bay, Ontario
P7E 2A3
(807)625-5100

Mr. Pat Houle
Espanola HS
Box 578
Espanola, Ontario
P0P 1C0
(705)869-1590

Mr. Walter W. Howard
Jarvis Collegiate Institute
495 Jarvis Street
Toronto, Ontario
M4Y 2G8
(416)393-0171

Mr. Gerald Hurtubise	Education Officer Ministry of Education Eastern Ontario Region 4th Floor, 1580 Merrivale Road Ottawa, Ontario K2G 4B5 (613)225-9210
Ms Margaret Johnston	Frontenac Secondary School 789 Bath Road Kingston, Ontario K7M 4Y3 (613)389-2130
Mr. John Klassen	Niagara Christian College 2619 Niagara Parkway Fort Erie, Ontario L2A 5M4 (416)871-6980
Mr. André Ladouceur	CSLF d'Ottawa-Carleton (catholique) Collège Catholique Samuel-Genest 675 rue Church Ottawa, Ontario K1K 3K1 (613)744-8344
Mr. Effie Lauferman	Community Hebrew Academy of Toronto 200 Wilmington Avenue Downsview, Ontario M3H 5J8 (416)636-5984
Mrs. Jeri Lunney	St. Paul's High School 39 Beaumaris Drive Nepean, Ontario K2H 7K5 (613)820-9705

Ms Jerry Maloney
Chelmsford Valley District Composite School
Box 249
Chelmsford, Ontario
P0M 1L0
(705)855-4594

Professor James Mayberry
Department of Mathematics
Brock University
St. Catherines, Ontario
L2S 3A1
(416)688-2789

Mr. Jim O'Connor
Chippewa Secondary School
539 Chippewa Street West
North Bay, Ontario
P1B 6G8
(705)472-4010

Mr. Chester Psica
Sir Sandford Fleming Secondary School
50 Ameer Avenue
North York, Ontario
M6A 2L3
416)787-4987

Mr. Reg Robinson
Iroquois Falls Secondary School
Box 369
Iroquois Falls, Ontario
P0K 1E0
(705)258-3921

Mr. Todd Romiens
Mathematics Co-ordinator
Windsor Board of Education
Box 210
451 Park Street
Windsor, Ontario
N9A 6K1
(519)255-3200

Mr. Geoff Roulet
Education Officer
Ministry of Education
Midnorthern Region
7th Floor, 199 Larch Street
Sudbury, Ontario
P3E 5P9
(705)675-4436

Mr. Donald Rousson
Confédération E. S.
670 Tanguay Avenue
Welland, Ontario
L3B 4G2
(416)732-1361

Mr. Michael Schefter
St. Michael's Secondary School
8 Grange Street
Stratford, Ontario
N5A 3P6
(519) 271-0890

Mr. Paul Zolis
Co-ordinator of Mathematics
Scarborough Board of Education
140 Borough Drive
Level 2
Scarborough, Ontario
M1P 4N6
(416)396-5614 or 396-6974

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